Finite Elements in Analysis and Design 44 (2008) 646 – 654

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Efficient finite element analysis by graph-theoretical force method; triangular and rectangular plate bending elements

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ARTICLE INFO

Article history:
Received 15 February 2007
Received in revised form 26 February 2008
Accepted 2 March 2008

Keywords:
Force method
Null basis
Graph theory
Algebraic methods
Plate bending
Finite elements

ABSTRACT

The calculation of null basis for equilibrium matrix is the main part of finite elements analysis via force method. For an optimal analysis, the selected null basis matrices should be sparse and banded corresponding to sparse, banded and well-conditioned flexibility matrices. There are many algorithms for the formation of null bases among which the algebraic methods benefit from the generality. However, the efficiency of these methods is highly dependent on the size of problems, and their computational times are very high for such problems. In this paper, a graph-theoretical method is presented for the formation of sparse, banded and highly accurate null basis matrices for finite element models with triangular and rectangular plate bending elements. These bases are generated much faster than those obtained by the algebraic methods. The efficiency of the present method is illustrated through some examples.

1. Introduction

The force method of structural analysis, in which the redundant forces are used as unknowns, is appealing to engineers, since the properties of members of a structure most often depend on the member forces rather than joint displacements. This method was used extensively until 1960. After this, the advent of the digital computer and the amenability of the displacement method for computation attracted most researchers. As a result, the force method and some of the advantages it offers in non-linear analysis and optimization has been neglected.

Four different approaches are adopted for the force method of structural analysis, classified as:

1. Topological force methods.
2. Algebraic force methods.
3. Mixed algebraic-combinatorial force methods.
4. Integrated force method.

Topological methods have been developed by Henderson [1], Maunder [2] and Henderson and Maunder [3] for rigid-jointed skeletal structures using manual selection of the cycle bases of their graph models. Methods suitable for computer programming are due to Kaveh [4–6]. These topological methods are generalized to cover different types of skeletal structures, such as rigid-jointed frames, pin-jointed planar trusses and ball-jointed space trusses [7,8].

Algebraic methods have been developed by Denke [9], Robinson [10], Topçu [11], Kaneko et al. [12], Soyer and Topçu [13] and mixed algebraic-topological methods have been used by Gilbert et al. [14], Coleman and Pothen [15,16], and Pothen [17]. The integrated force method has been developed by Patnaik [18,19], in which member forces are used as variables, the equilibrium equations and the compatibility conditions are satisfied simultaneously in terms of these variables.

Graph-theoretical methods for the formation of sparse and banded null bases for models consisting of triangular plane stress and strain elements, tetrahedron elements, and the inclusion of the effect of indeterminate support conditions are developed by Kaveh and Koohestani [20–22]. Graph theory is also utilized for hybrid elements in the modelling of plates, Maunder [23].

In this paper, an efficient algorithm is presented for the formation of null bases for finite element models consisting of triangular and rectangular plate bending elements. The null bases obtained by this algorithm are highly sparse and narrowly banded and can be used for optimal finite element analysis by force method. In the present method, using topological transformations the non-zero patterns of null bases are identified and their numerical values are calculated by an algebraic process. For this purpose, associate digraph and interface graph are defined and utilized. These calculations are restricted to submatrices with small sizes which lead to substantial saving in computational time as well as high accuracy.

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doi:10.1016/j.finel.2008.03.001