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ScienceDirect
Finite Elements in Analysis and Design 43 (2007) 543–554
FINITE ELEMENTS IN ANALYSIS AND DESIGN
www.elsevier.com/locate/finel

Efficient finite element analysis by graph-theoretical force method

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Received 22 March 2006; received in revised form 29 September 2006; accepted 4 December 2006
Available online 24 January 2007

Abstract

In this paper an efficient method is developed for the formation of null bases of triangular plane stress and plane strain finite element models, corresponding to highly sparse and banded flexibility matrices. This is achieved by associating special graphs to the finite element models and selecting appropriate subgraphs for the formation of localized self-stress systems. The efficiency of the present method is illustrated through some examples.

Keywords: Finite elements; Graph theory; Null basis; Flexibility matrix; Sparsity; Ordering

1. Introduction

The force method of structural analysis, in which the member forces are used as unknowns, is appealing to engineers, since the properties of members of a structure most often depend on the member forces rather than joint displacements. This method was used extensively until 1960. After this, the advent of the digital computer and the amenability of the displacement method for computation attracted most researchers. As a result, the force method and some of the advantages it offers in nonlinear analysis and optimization has been neglected.

Four different approaches are adopted for the force method of structural analysis classified as:

1. topological force methods;
2. algebraic force methods;
3. mixed algebraic-combinatorial force methods;
4. integrated force method.

Topological methods have been developed by Henderson [1] and Maunder [2] for rigid-jointed skeletal structures using manual selection of the cycle bases of their graph models. Methods suitable for computer programming are due to Kaveh [3–5]. These topological methods are generalized to cover different types of skeletal structures, such as rigid-jointed frames, pin-jointed planar trusses and ball-jointed space trusses [6,7].

Algebraic methods have been developed by Denke [8], Robinson [9], Topçu [10], Kaneko et al. [11], Soyer and Topçu [12] and mixed algebraic-topological methods have been used by Gilbert and Heath [13], Coleman and Pothen [14,15], and Pothen [16].

The integrated force method has been developed by Patnaik [17,18], in which the equilibrium equations and the compatibility conditions are satisfied simultaneously in terms of the force variables.

In this paper an efficient method is developed for the formation of null bases of triangular plane stress and plane strain finite element models, corresponding to highly sparse and banded flexibility matrices, This is achieved by associating a special graph to the finite element model, and selecting subgraphs (known as γ-cycles [19]) for the formation of localized self-stress systems (null vectors). The efficiency of the present method is illustrated through simple examples.

2. Algebraic force methods

Consider a discrete or discretized structure S, which is assumed to be statically indeterminate. Let $\mathbf{r}$ denote the m-dimensional vector of generalized independent element

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0168-874X/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.finel.2006.12.002