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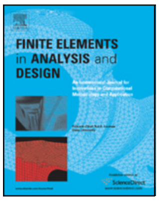

A general theorem for adjacency matrices of graph products and application in graph partitioning for parallel computing

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Many regular models can be viewed as the graph products of two ore more subgraphs know as their generators. In this paper, a general theorem is presented for the formation of adjacency matrices using a series of algebraic relationships. These operations are performed on the adjacency matrices of the generators. The Laplacian matrix of the graph product is then formed and the second eigenvalue and the corresponding eigenvector are used for the bisection of the regular graphs associated with space structures or finite element models.

1. Introduction

Graph theory has been successfully applied to many problems in computational mechanics, Kaveh [1,2]. Graph products, are employed in configuration processing, node/element ordering and partitioning of regular space structures and finite element models. These products are also employed in easy calculation of eigenvalues, and employed in calculation of buckling load and eigenfrequencies of regular structures. A structure is called *regular* if it can be generated by the product of two or more subgraphs.

The data structure of a graph can be represented by different means. The adjacency matrix is one such a representation often used in algebraic graph theory. Algebraic graph theory is a branch of graph theory, where eigenvalues and eigenvectors of adjacency and Laplacian matrices are employed to deduce the principal properties of a graph. In fact eigenvalues are closely related to most of the invariants of a graph, linking one extremal property to another. These eigenvalues play a central role in our fundamental understanding of graphs. There are interesting books on algebraic graph theory such as Biggs [3], Cvetković et al. [4], and Godsil and Royle [5].

One of the major contributions in algebraic graph theory is due to Fiedler [6], where the properties of the second eigenvalue and eigenvector of the Laplacian of a graph have been introduced. This eigenvector, known as the *Fiedler vector* is used in graph nodal ordering and bipartition, Refs. [7–9].

General methods are available in literature for calculating the eigenvalues of matrices, however, for matrices corresponding to special models, it is beneficial to make use of their extra properties. These methods have many applications in computational mechanics, such as ordering, graph partitioning, and subdomaining finite element models, Kaveh and Rahami [10,11].

At the beginning of the seventh decade of this century special graph products, known as the *non-complete extended P-sum* (NEPS), were introduced by Cvetković [4]. Generalization of these products can be found in the work of Sokarovski [12]. In this paper, the formation of adjacency matrices is studied. More general definitions are introduced for graph products than NEPS, and then a theorem is proved which can be used to find the properties of the adjacency matrices of the existing graph products. Once the adjacency matrix for a graph product is formed, then the Laplacian matrix for this graph can be constructed. The second eigenvalue of this matrix is calculated and the corresponding eigenvector is used for the bisection of the finite element models.

2. Graph products

2.1. Basic definitions from graph theory

A graph S consists of a set $N(S)$ of elements called nodes (vertices or points) and a set $M(S)$ of elements called members (edges or arcs) together with a relation of incidence which associated with each member a pair of nodes, called its ends. The connectivity properties of a skeletal structure can simply be transformed into that of a graph S; the joints and the members of the structure correspond

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