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Combinatorial optimization of special graphs for nodal ordering and graph partitioning

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Abstract There are numerous applications of graph theory and algebraic graph theory in combinatorial optimization and optimal structural analysis. In this paper, a new canonical form as well as its relation with four structural models often encountered in practice and their corresponding graphs are presented. Furthermore, the block diagonalization of this form, which is performed using three Kronecker products and unsymmetric matrices, is studied. This block diagonalization leads to an efficient method for the eigensolution of adjacency and Laplacian matrices of special graphs. The eigenvalues and eigenvectors are used for efficient nodal ordering and partitioning of large structural models. The present method is far more simple than any existing general approach.

1 Introduction

There are several applications of eigenproblems in structural mechanics. The eigenvalues and eigenvectors of graphs play an important role in algebraic graph theory and combinatorial optimization. Large structural models and their corresponding graphs have large and sparse matrix representations. The factorization of these matrices with arbitrary patterns requires general methods. However, there are some structural models with special topologies the associated matrices of which can be transformed to particular patterns in such a manner that their factorizations can more easily be performed. Some of these matrices were previously studied by Kaveh and Sayarinejad [1,2], Kaveh [3], Kaveh and Rahami [4], and some other general eigensolution methods are also available in Robbe and Sadkane [5], Park [6], Mathias and Stewart [7] and Hasan and Hasan [8], among many others.

There are also some applications of graphs spectra in nodal and element ordering and graph partitioning. Further information about these issues can be found in the work of Gould [9], Straffing [10], Maas [11], and Grimes et al. [12].

Paulino et al. [13,14] have applied the second eigenvalue and eigenvector of the Laplacian matrix of a graph, known as Fiedler vector, to element and nodal numbering [15]. Mohar [16], Pothen et al. [17], Simon [18], Seal and Topping [19], Barnard et al. [20], Kaveh and Rahimi Bondarabady [21], Kaveh [3] used the properties of the Fiedler vector for ordering and graph partitioning problems. Other applications related to graph products and regular structures were also previously studied by Kaveh and Rahami [4,22].

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