

## INVESTIGATION OF SEISMIC PERFORMANCE OF STEEL FRAMES BASED ON A QUICK GROUP SEARCH OPTIMIZER

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### ABSTRACT

A quick group search optimizer (QGSO) is an intelligent optimization algorithm which has been applied in structural optimal design, including the hinged spatial structural system. The accuracy and convergence rate of QGSO are feasible to deal with a spatial structural system. In this paper, the QGSO algorithm optimization is adopted in seismic research of steel frames with semi-rigid connections which more accurately reflect the practical situation. The QGSO is combined with the constraint from the penalty coefficients and dynamic time-history analysis. The performance of the QGSO on seismic design has been tested on a two-bay five-layer steel frame in this paper. The result shows that, compared with the PSO algorithm, the QGSO algorithm has better performance in terms of convergence rate and the ability to escape from local optimums. Moreover, it is feasible and effective to apply the QGSO to the seismic optimal design of steel framework.

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**KEY WORDS:** quick group search optimizer; semi-rigid connection; seismic optimal design; discrete variables; time history analysis.

### 1. INTRODUCTION

Recently, the steel structures have been widely applied in bridges, industrial plants and stadia due to the properties of light weight, environment-friendly characteristics, flexible forms and easy construction. However, the applications of the steel structures are hindered because of the high construction cost. The optimization of the steel structures has been proposed to promote their applications. As the computational technology is widely used and

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the optimization become more satisfactory, new optimization methods have been proposed to find the best effect of structural sections, shapes and distribution in the structural field.

This paper has applied a quick group search optimizer (QGSO) to seismic design optimization of a two-bay five-layer steel frame, the optimization aims to find the lowest weight of the steel structure with semi-rigid joints which must meet the seismic requirements. In the design of the steel structures, the joints are always treated as rigid connection, which has the advantage in reducing the difficulty of calculation and design, but this simplification does not reflect the practical situation. The ideal rigid joints and hinge points are unreal; the stiffness of all joints is between the rigid connection and the fully articulated. As a result, the semi-rigid connection has been increasingly important in modeling of the steel and concrete structures. The investigations on the steel structures with semi-rigid connection showed that many joints demonstrate the semi-rigid characteristics [1-4]. A large number of tests had been carried out to measure the force and displacement of the semi-rigid joints. Vimonsatit *et. al* [5] had a hysteresis study of steel structures with semi-rigid connections. Nguyen [6] had conducted the nonlinear elastic dynamic analysis of spatial steel frames with semi-rigid connections. An optimization design method for nonlinear steel frames with semi-rigid connections and semi-rigid column bases using a genetic algorithm was proposed by Hayalioglu [7]. Shi et al [8, 9] had numerical study of the hysteresis performance of semi-rigid joints in the steel structures. Modeling semi-rigid connections has brought a great deal of difficulty comparing with the traditional joint models, especially in convergence and efficiency. Numerical simulations of the dynamic performance of the steel structures with semi-rigid joints would be a hot issue in the future.

There are several algorithms that have been adopted in optimal design of steel framework such as GA algorithm [9, 10], PSO algorithm [11], ACO algorithm [12] etc. Among them, the GA algorithm has the fast stochastic global search capability, but it cannot effectively use the system feedback when the solution reaches a certain extent. A lot of redundant iterations are run by the GA algorithms, hence it is inefficient to find the best solution. The PSO algorithm has been applied to the structural optimization field for a while. It is easy to fall into a local optimal solution because it is imperfect and unsatisfactory. The ACO algorithm updates pheromone accumulation and converges to the optimal path. It has a distributed parallel global search capability, but lacks the initial pheromone, which leads to a slow solution. The QGSO, which has been derived from the group search optimizer algorithm (GSO) [13], will be described in this paper. It is used in the seismic resistance design optimization of steel frame and compared with the PSO algorithm. In order to obtain a more efficient method, the QGSO has made use of the step search method from the PSO algorithm and abandon the angle search. Thus the QGSO is more computationally efficient for the seismic design that needs a large amount of computations. Meanwhile, the capacity of escaping from the local minimum is the weak point of the PSO algorithm. The QGSO algorithm combines with GA to generate the rogue variation at the same time, which enhances the ability of escaping from local minimums.

In recent years, many academic researchers have studied the structural optimization design problems without considering the seismic loads completely. They treated the dynamic loads as the static loads; there is a limited usage of history analysis because of its high time-consumption. But the method of the dynamic history analysis adopted in

engineering could better simulate the real situation of the earthquake impact on the structures. Most scholars adopt GA and ACO algorithm which are more complex to carry out seismic optimization design. The QGSO has an easy theory and fewer parameters, which has a good performance in many large optimization projects, as its convenient usage and quick convergence.

The QGSO algorithm had successful applications in truss static optimization, which has been adopted in steel structures seismic optimization design firstly now, and the results shows that the QGSO algorithm is accurate and converges quickly. It also has better performance in escaping from local optimum compared with the PSO algorithm.

## 2. A QUICK GROUP SEARCH (QGSO) OPTIMIZATION

The quick group search optimization algorithm QGSO [14] is based on the GSO algorithm with the same classification model of the behavior among the group members to randomly search in an  $n$ -dimensional search space. The  $i$ th member at the  $k$ th search iteration has a current position, which is randomly initialized before the iterative process begins. After  $k$  iterations, for each member the fitness values will be calculated and the best fitness value will be chosen as the producer whose position is made of  $X_{Gbest}^k$ . The rest of the members with a certain probability randomly turn into the scrounger and approach to the producer by a stochastic step. The formula is as follows:

$$X_i^{k+1} = X_i^k + w_1 \times r \otimes (X_{Gbest}^k - X_i^k) + w_2 \times r \otimes (X_{i,Pbest}^k - X_i^k) \quad (1)$$

where  $r$  is an  $n$ -dimensional vector that describes a random sequence in the range (0, 1);  $w_1$  and  $w_2$  represent the information transfer factors;  $X_{i,Pbest}^k$  stands for the best previous position of the  $i$ th particle. Combining the genetic algorithm, the rangers exchange information with the producer by a certain probability. The scroungers move toward the producer, not only adopting the message of a producer but also considering their own information. The remaining members are the rangers searching for the next producer deliberately.

To handle the geometric constraint boundary, the method introduced in HPSO algorithm [15] was utilized in the paper. The HPSO algorithm combines the PSO algorithm and harmony search algorithm for constraint handling principles i.e. when the vectors of a particle fly out from the range of variables, their spatial position will be regenerated.

## 3. FITNESS FUNCTION AND THE DISCRETE VARIABLES

### 3.1. Revision model of the fitness function

The exterior penalty function method [16] is widely utilized in engineering optimization design, therefore the idea of the penalty function was introduced into the following expression to constrain particles which violate the performance constraints:

$$F(X_i, r_i) = Obj_i + (r_1 \times \max(g_1(X), 0) + r_2 \times \max(g_2(X), 0)) \quad (2)$$

where  $r_1$  and  $r_2$  are the penalty balance coefficient;  $i=1,2,\dots,N$ ,  $F(X_i, r_i)$  represents the fitness function value at the  $i$ th particle;  $g_1(X)$ ,  $g_2(X)$  are the two constraint functions about the  $i$ th particle that corresponding objective function value is ( $Obj_i$ ).

To ensure the optimal solution fall in the feasible region, in this paper, the constraint violation balance coefficient  $r_i$  is introduced to adjust the sensitivity of a constraint violation, making the different nature of the constraint have varying degrees of punishment. The various constraint magnitude is balanced by setting the  $r_i$  parameter, which contributes to the fitness value with the same sensitivity.

### 3.2 The solution of the discrete variables

The QGSO algorithm optimizes the size of non-continuous sections, and the mapping function, which makes the discrete section areas correspond to the continuous integer according to the control factor in ascending sort, is set before optimization. A discrete set  $A_n$  in ascending order with  $n$  discrete variables can be expressed as follows:

$$A_n = \{X_1, X_2, \dots, X_j, \dots, X_n\}, \quad 1 \leq j \leq n \quad (3)$$

The example in the paper is a steel frame. Due to the characteristics of the structure, the main control factors are based on the moment of inertia for the beams and the section areas for the columns, therefore the beams and columns of the steel frame are divided into two groups by using a mapping function, respectively. The mapping function uses the serial number instead of discrete values of  $A_n$  and makes the value continuous and avoids inefficiency. Suppose that there are  $p$  members in the search space with  $D$  dimension and the position of the  $i$ th member is denoted with vector  $x_i$ :

$$x_i = (x_i^1, x_i^2, \dots, x_i^d, \dots, x_i^D), \quad 1 \leq d \leq D \quad i = 1, \dots, p \quad (4)$$

where  $x_i^d \in \{1, 2, \dots, j, \dots, n\}$  corresponds to the discrete variables  $\{X_1, X_2, \dots, X_j, \dots, X_n\}$  by the mapping function  $h(j)$ . Hence the entire members search in the continuous integer space and each component of vector  $x_i$  is an integer. Then the continuous values randomly generated can be dealt with by employing the round-up method to ensure the safety of the structure.

## 4. OPTIMIZATION PROCESS OF THE ASEISMIC DESIGN OF STEEL FRAMEWORK

The following section shows the optimization process of the aseismic design of the semi-rigid steel frame:

- 1) Data Preparation: set geometrical parameters, material parameters and algorithm parameter, fulfill the steel section table.
- 2) Population initialization: generate randomly initial population; combine discrete steel section table with a set of mapping functions; arrange beam cross sections and column sections in ascending sort according to the moment of inertia and section area respectively;
- 3) Minimize the weight of the structure with the stress and displacement constraints;
- 4) Introduce a balanced parameter to adjust the fitness value; take advantage of back-flying technology to deal with the cross-border particles and update the population;
- 5) Stop if the maximum iterations reached.

### 5. NUMERICAL EXAMPLES

#### 5.1 Description of numerical examples

Figure 1 shows a two-bay five-layer steel frame model derived from the reference [17].

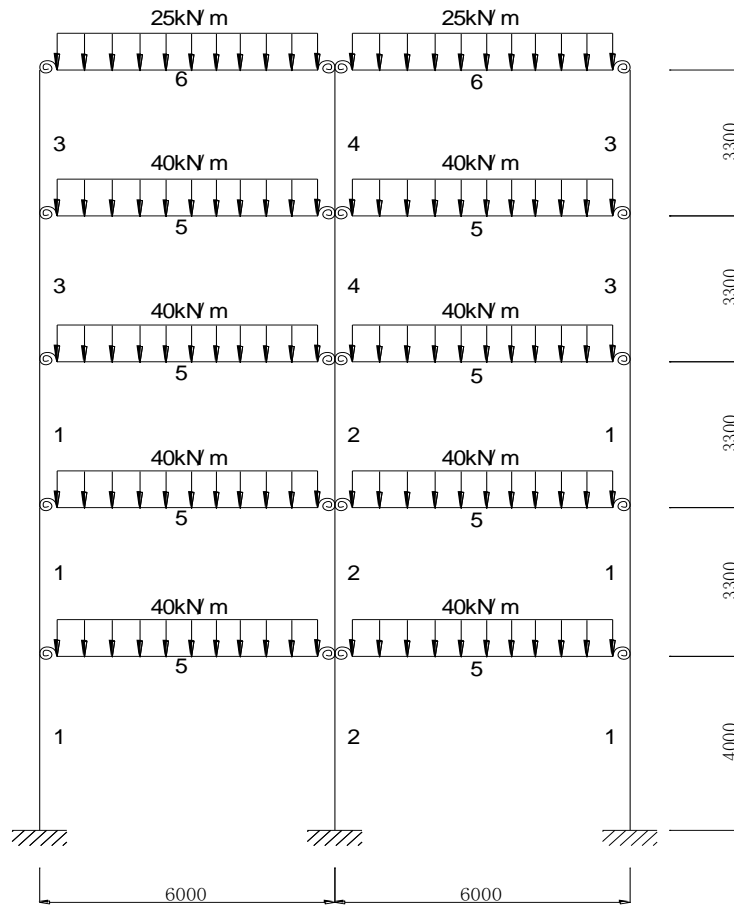


Figure 1. Two-bay five-layer plane steel frame with semi-rigid joints

The material of the steel frame is Q235, of which the Young's modulus is  $2.10 \times 10^{11}$  N/m<sup>2</sup> and the mass density 7850 kg/m<sup>3</sup>. The beams are selected from the HN standard sections and I-beam cross-sections [18] and the frame columns are selected from the HW, HM standard sections [19]. The self weight of the structure was taken into account. The line load 25 kN/m is applied on the steel beam of the top floor and 40 kN/m is applied on the rest of beams. The total weight of the structure meets the standard of the structural integrity requirements of aseismic design.

In this paper, a more precise simulation model of the beam-column joints has been taken into account, as represented by the two-web-flange and top-seat connections in Figure 2. The web is designed to connect angles, which increase the joint stiffness. The force-displacement relation of semi-rigid connection under the actual loading process is generally nonlinear, which is influenced by many factors such as connection form, cross-sectional area, bolt sizes, the connecting end plates or angle sizes, etc. The spring unit (COMBIN39) element in ANSYS was used to simulate the semi-rigid connection. The spring unit used the nonlinear constitutive law proposed by Colson and Louvoau [20], which can be described by the Eq. (5). The spring unit allows the relative rotation, but not the relative translation, between the beam and column connected by it. The generalized force-deformation curve has been input into ANSYS to simulate the nonlinear characteristics of the semi-rigid joints as follows:

$$\theta_r = \frac{M}{R_{ki}} \frac{1}{\left[ 1 - \left( \frac{M}{M_u} \right)^n \right]} \quad (5)$$

where  $R_{ki}$  is the initial connection stiffness;  $M_u$  is the ultimate moment capacity of connection;  $n$  is the parameter which represents the shape of the curve ( $M - \theta_r$ ). The initial stiffness and ultimate moment were selected from the experimental data of two-web-flange and top-seat connections which was studied by Wang [21], from which the 15 sets of data were employed for a constitutive model.

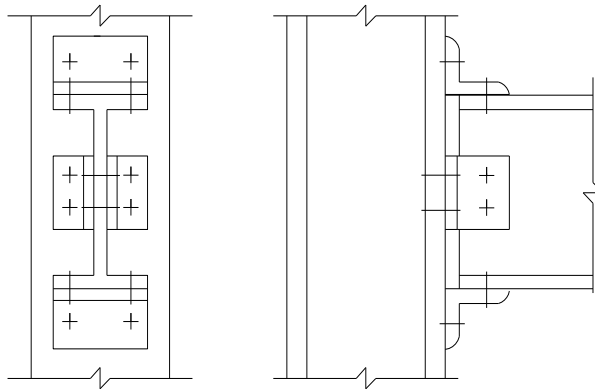


Figure 2. Two-web-flange and top-seat connections of beam-column joints

## 5.2 Aseismic analysis

The structural model was utilized in the dynamic history analysis based on the original seismic data (EL-Centro: The peak acceleration of north-south direction is 0.34g). The peak acceleration, added to the bottom of the structure, has been adjusted by the seismic design code, and the time step  $\Delta t = 0.02$  sec was determined to calculate a 20 seconds of earthquake response.

## 5.3 Mathematical model

### 5.3.1 Geometric limits

The design variables are selected from hot-rolled H and cut T section steel regulated by the Chinese Standard (GB T11263-2005) [19]. All components use the Q235 steel. Totally 27 HW sections and the HM sections from GBT11263-2005 were assigned to columns, as well as 33 HN sections from the GB T11263-2005 and the hot-rolled I-beam from GB/T706-1988 [18] assigned to frame beams. The upper and lower limits of the design variables are set as follow:

$$\text{LowerBound} = \text{ones}(\text{NDim}, 1); \quad \text{UpperBound} = K \times \text{ones}(\text{NDim}, 1) \quad (6)$$

where *LowerBound* and *UpperBound* are the upper and lower bounds of the design variables respectively, whose dimensions are  $\text{NDim} \times 1$ ; The matrix *ones* ( $\text{NDim}, 1$ ) produces an array of values which all equal to 1; The constant *K* represents the beam and column limit values which can be adjusted according to the dimension of the practical applications; where  $K \times \text{ones}(\text{NDim}, 1)$  means that the constant *K* is expanded into an array (the upper bounds of the design variables).

Meanwhile, to satisfy the construction requirements, the flanges of the steel beams are smaller than the flanges of the steel columns.

### 5.3.2 Constraint limits

To consider aseismic performance of the steel frame structures, the design stress is adjusted by seismic adjustment coefficient of bearing capacity such as the following inequality (7):

$$\frac{N_k}{A_k} \pm \frac{(M_k)_x}{r_x (W_k)_{nx}} \pm \frac{(M_k)_y}{r_y (W_k)_{ny}} \leq \frac{f}{r_{RE}}, \quad r_{RE} = 0.75, \quad 215 \text{MPa} / 0.75 = 286.7 \text{MPa} \quad (7)$$

where  $A_k$  stands for the cross-sectional area of the member;  $\frac{(M_k)_x}{r_x (W_k)_{nx}}$  and  $\frac{(M_k)_y}{r_y (W_k)_{ny}}$  are the stress values about the strong axis and weak axis of the cross-section with the plastic development coefficient, respectively; *f* is the design value for the strength of steel and  $r_{RE}$  is the seismic adjustment coefficient.

At the same time, the stability of the whole structure should be considered, which could

be described by the formula 4.2.2 in the reference [22].  $\frac{M_x}{\phi_b W_x} \leq f$  ensures the overall stability of the steel structure, in which  $\phi_b$  is the coefficient of the overall stability of the beam.

The seismic resistant in appendix M.1.3 of the design code GB50011-2010 [23] describes that the aseismic performance considering the layer displacement as a control target when the vertical components experience different damage level. In this example the inter-story displacement angle  $\theta$  was set to 1/300 to indicate that is in a perfect condition. It means that the vertex of the structure takes the most unfavorable displacement limit:  $u_{\max} = 0.057m$ .

### 5.3.3 Mathematical model

The mathematical model of weight optimization:

$$\min W = \sum_{i=1}^n \rho_i A_i l_i \quad (8)$$

The mathematical model of the steel section discretization:

$$A_n = \{X_1, X_2, \dots, X_j, \dots, X_n\}, \quad 1 \leq j \leq n \quad (9)$$

The mathematical model of structural performance and geometric constraints:

$$\begin{aligned} g_k(X) &\leq \frac{f}{r_{RE}} & k = 1 \\ g_k(X) &\leq f \times \varphi & k = 2 \\ g_k(X) &\leq u_{\max} & k = 3 \\ A_i &\in [A_1 \quad A_2 \quad \dots \quad A_n]^T \end{aligned} \quad (10)$$

where  $n$  corresponds to the number of components;  $A_i$  is the area of the  $i$ th member;  $X_j$  represents the section discretization number;  $A_n$  is the discrete-section set of variables;  $\rho_i$  stands for the mass density of the material;  $l_i$  is the length of the component;  $g_1(x)$  represents the constraint functions for structural strength;  $g_2(x)$  represents the constraint functions for the stability;  $g_3(x)$  is regarded as the constraint functions for the most unfavorable displacement.

## 6. RESULTS AND DISCUSSION

The stress and the displacement constraints were considered in this paper. Numerical



calculations results in Figure 3 show that the algorithm is very stable. The program run 40 iterations and most of results converge quickly which proves that the QGSO is feasible and effective to solve aseismic optimization design of the steel frames. Table 1 shows the optimized results of the structural weight.

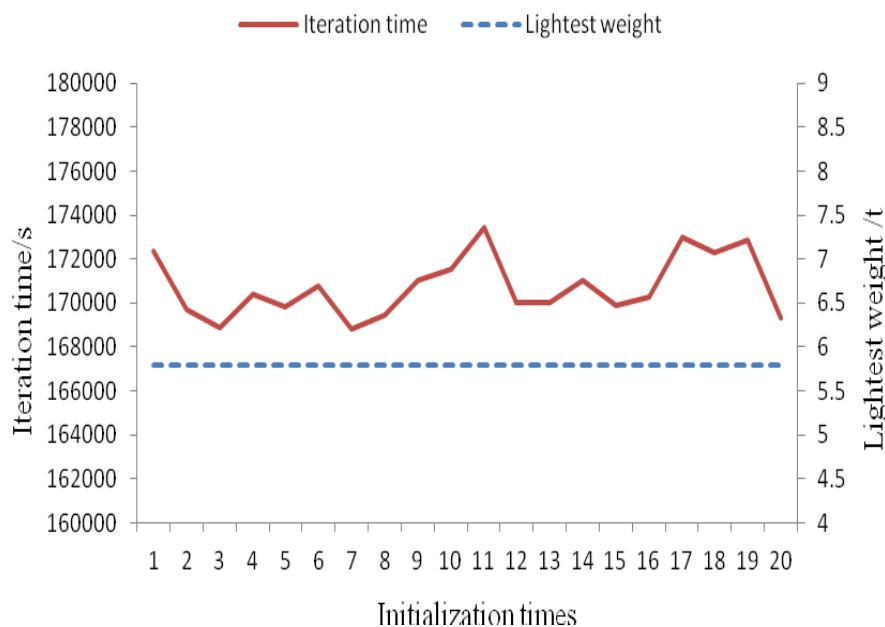


Figure 3. Iteration times and the lightest weight

Table 1: Comparison of the optimization results for two-bay five-layer steel frame

Population	Un-optimized /ton	Optimized/ton	Time/h	Iterations
20	8.674	5.7872	48	40

To prove the correctness of the program, the optimization results are transferred to the finite element software to verify. The lateral displacement of the vertex constraints under the aseismic performance of the structures has been checked in the paper. If the lateral displacement is less than the maximum constraint,  $u_{max} = 0.057$  m, the structure is in perfect condition. Figure 4 reflects that the maximum displacement of the un-optimized structure may reach 0.0172 m, but the maximum displacement of optimized frame reduced to 0.0126 m and the weight of structure reduced 2.89 t as well, while the horizontal displacement is much less than the prescribed limits, which has satisfied the standard requirements for the architectural integrity of the aseismic performance. Figure 5 shows that the QGSO finds the best section for the frame beams and columns during the early stage of the process.

To compare with the literature [17], the optimal results of the numerical examples are presented in the Table 2. Introducing the ordinary I-beam sections into steel table, which better simulates the practical structures, the optimized weight is smaller than that in [17], which only used the hot-rolled H-beam sections for optimization.

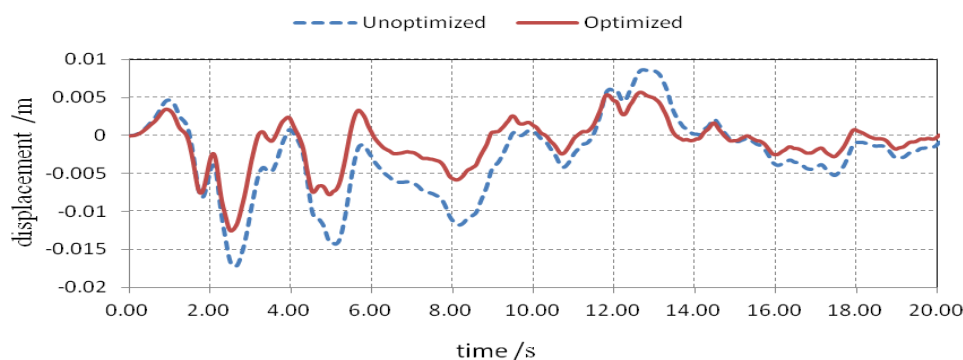


Figure 4. The maximum horizontal displacement for un-optimized and optimized frame

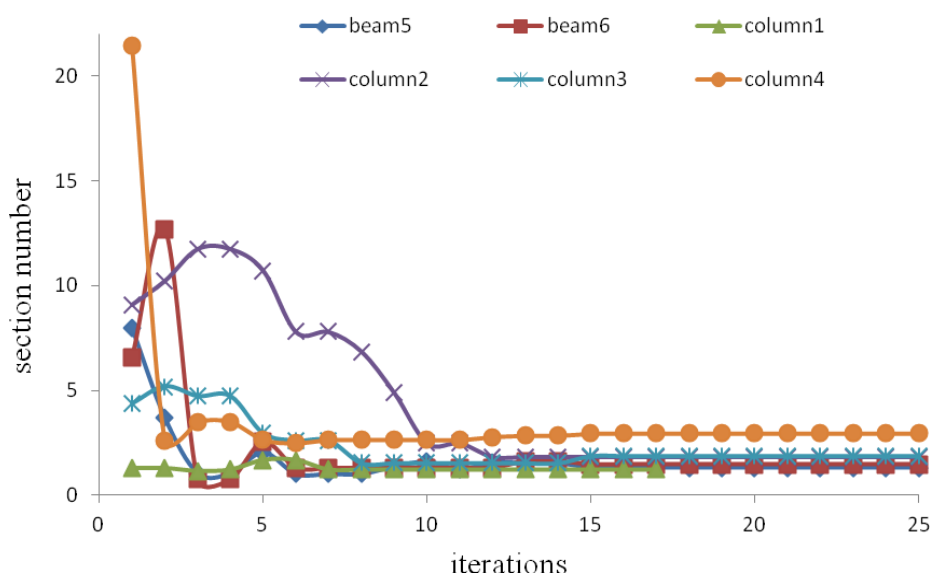


Figure 5. Iterative optimization process of the column sections and beam sections of frame

Table 2: Comparisons of optimization results for the steel sections

Type	Section number	Section size (QGSO)	Section size (GA) [17]
Frame columns	1	HM340×250×9×14	H350×300×8×14
	2	HM340×250×9×14	H400×300×10×16
	3	HM340×250×9×14	H340×250×8×12
	4	HM340×250×9×14	H340×250×10×12
Frame beams	5	HN298×149×5.5×8	H450×150×8×10
	6	20a 200×100×7×11.4	H400×150×8×10
Weight / t		5.7872	8.2102

Note: The HM means that the width of flange is middle level and the HN stands for narrow flange. The '20a' represents the type of Hot rolled I-beam, in which the '20' means the height of the cross section and the 'a' is a label which corresponding different width of cross section and thickness of the web.

## 7. COMPARISON OF PARTICLE SWARM OPTIMIZATION

Based on the comparisons between QGSO and PSO algorithms, it is easy to see the PSO algorithm is more easily to fall into a local optimum. For example, there is a case that runs 100 iterations using the PSO algorithm and the QGSO algorithm respectively in this paper. At the 20-th iteration, the PSO algorithm stuck into a local optimum at 6.5644 t, while the QGSO escaped from the local optimum and found the better value of 5.7872 t. After many tests, it shows that PSO algorithm is more easily falls into the different local optimums which it cannot go through. It is undeniable that the QGSO algorithm could be adopted into the optimization of the structural aseismic design, which has a good convergence rate and accuracy as well as good stability. Therefore it is possible to apply the QGSO algorithm to optimization design of the structural aseismic performance.

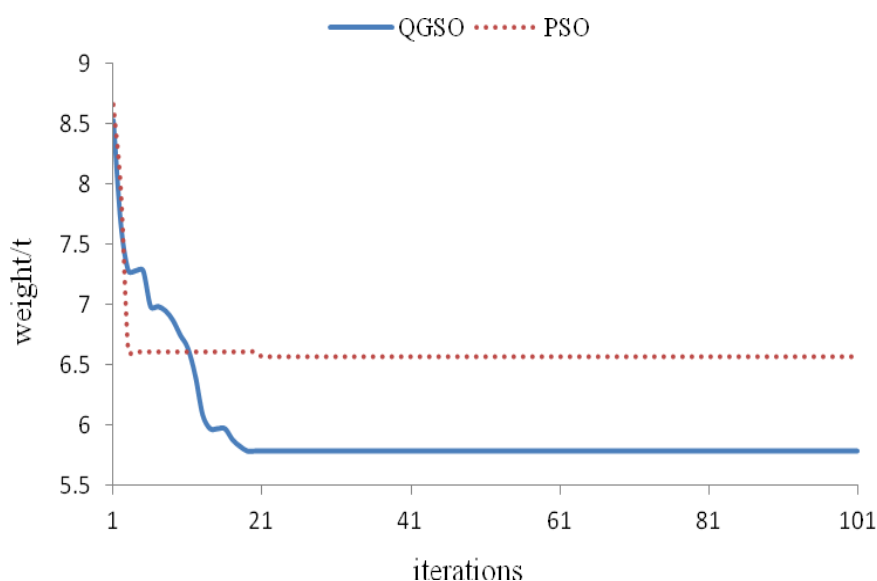


Figure 6 Convergence rates between the QGSO and the PSO

## 8. CONCLUSION

In this paper, the investigation of a quick swarm optimization algorithm is first applied to the aseismic optimization design of steel frames with semi-rigid joints. The numerical examples calculation results shows:

The QGSO algorithm, which is firstly applied in optimization of the steel-structure aseismic design with semi-rigid joints, has a better convergence rate and accuracy than the PSO algorithm. The QGSO algorithm is easy to escape from local optimal values (further reduce the weight about 1.2 ton).

A more accurate structural model was introduced: the dynamic time-history method takes a long time, as it accurately simulates the seismic loads on structures; the applications of the semi-rigid beam sections better represent the actual situation;

Considering both geometric and performance constraints, it demonstrated that QGSO algorithm could be applied to the optimizations of the structural aseismic design.

At last, considering the geometric and performance constraints at the same time, the numerical results confirmed that the QGSO algorithm can be applied to seismic optimization studies. It has the potential to be employed in the practical engineering projects where a large amount of calculations are expected.

In summary, the QGSO algorithm can be applied to aseismic design of steel frames with semi-rigid connections and provide designers a useful guideline.

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