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STABILITY BASED OPTIMUM DESIGN OF CONCRETE GRAVITY DAM USING CSS, CBO AND ECBO ALGORITHMS

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ABSTRACT

This study presents shape optimization of a gravity dam imposing stability and principal stress constraints. A gravity dam is a large scale hydraulic structure consisting of huge amount of concrete material. Hence, an optimum design gives a cost-benefit structure due to the fact that small changes in shape of dam cross-section leads to large saving of concrete volume. Three recently developed meta-heuristics are utilized for optimizing the structure. These algorithms are charged system search (CSS), colliding bodies optimization (CBO) and its enhanced edition (ECBO). This article also provides useful formulations for stability analysis of gravity dams which can be extended to further researches.

Keywords: gravity dam; structural optimization; stability analysis of dam; charged system search; colliding bodies optimization, meta-heuristics.

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1. INTRODUCTION

Structural design optimization can be categorized to three types including size optimization, shape optimization and topology optimization. Each one is used for a different scope. However, an objective function is usually selected so that an economical design is gained. Structural optimization plays much important rule for large-scale structure such as towers, dams, tall building and so forth. Meanwhile, gravity dam is a hydraulic structure constructed

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by rigid materials and the external actions imposed on the gravity dam are sustained by its own weight. Main positive features of gravity dams are summarized as they have: (I) simple design procedure; (II) no significant height limitation; (III) failure with adequate warning; and (IV) minimal maintenance requirements.

Concrete gravity dam is a well-known hydraulic structure constructed across the river valley to impound the water for social, energy and economical purposes. The economy and safety of the structure are the key points in design. Many attempts have been made for optimal design of dams, namely, arch and gravity dams. Khatibinia and Khosravi [1] performed shape optimization of gravity dam using gravitational search algorithm considering fluid-structure interaction. Kshirsagar [2] investigated the effect of variation of earthquake intensity on stability of Tilari gravity dam located in Maharastra, India according to Indian design criteria [3]. Salmasi [4] employed genetic algorithm for design optimization of gravity dam. Deepika and Suribabu [5] used differential evolution algorithm for optimal design of concrete gravity dam [2] based upon the Indian design criteria. There are also numerous researches on optimum design of arch dams [6-8]. Recent advances and main points on arch dam optimization are appropriately written in Ref. [9].

Meta-heuristics have overcame to many types of structural design optimization problems such as trusses, steel and reinforced concrete frames, dams, towers, reinforced concrete dual systems and so on [1, 6-8, 10-15]. Therefore, the experiences demonstrated their capability for extremely nonlinear optimization. This study presents shape optimization of a gravity dam imposing stability and principal stress constraints. The dam was already analyzed by Kshirsagar [2] and was then optimized by Deepika and Suribabu [5] as mentioned before. Here, three recently proposed meta-heuristics are applied to optimize this structure. These physics-inspired algorithms are charged system search (CSS) [16], colliding bodies optimization (CBO) [17] and enhanced colliding bodies optimization (ECBO) [18]. Furthermore, the current study provides useful formulations for stability analysis of gravity dams which can be utilized for development. Some comments on the optimization problem are also presented.

2. STABILITY ANALYSIS OF GRAVITY DAM

In this section, forces acting on the gravity dam, Fig. 1, are depicted in Tables 1 and 2. These calculations are derived based on a case study solved in [2] and the design criteria [3].

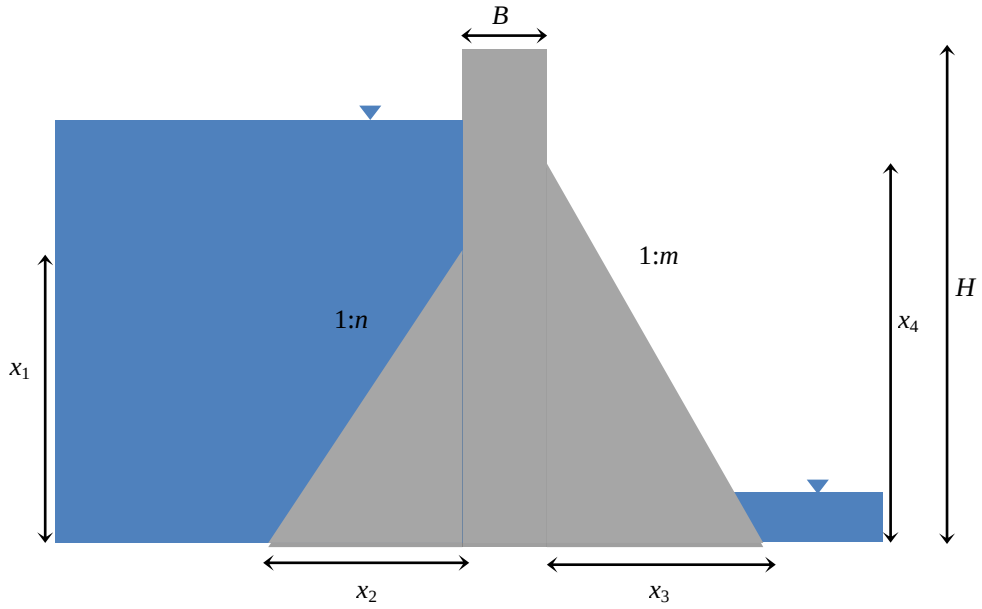

Figure 1. Schematic of a gravity dam and its shape parameters.

Part of these forces contribute to overall overturning moment and the other part contributes in overall resisting moment. The forces contributing to overturning moment are usually shown with mines sign. Table 3 defines the parameters utilized in the dam analysis. For more details, one can refer to the above mentioned references.

Table 1: Computation of vertical forces acting on the gravity dam.

No.	Category	Vertical force	Liver arm about toe
1	Self-weight	$W_1 = \frac{1}{2}\gamma_c x_1 x_2$	$x_3 + B + \frac{1}{3}x_2$
2		$W_2 = \gamma_c BH$	$x_3 + \frac{B}{2}$
3		$W_3 = \frac{1}{2}\gamma_c x_3 x_4$	$\frac{2}{3}x_3$
4		$P_{V_1} = \frac{1}{2}\gamma_w x_1 x_2$	$x_3 + B + \frac{2}{3}x_2$
5		$P_{V_2} = \gamma_w x_2 (h - x_1)$	$x_3 + B + \frac{1}{2}x_2$
6		$P_{V'} = \frac{1}{2}\gamma_w (mh')h'$	$\frac{mh'}{3}$
7	Uplift pressure force	$U_1 = \frac{1}{3}\gamma_w (x_2 + d_g)(h - h')$	$x_3 + B + \frac{1}{3}(2x_2 - d_g)$
8		$U_2 = \frac{1}{3}\gamma_w (x_2 + d_g)(h + 2h')$	$x_3 + B + \frac{1}{2}(x_2 - d_g)$

9		$U_3 = \frac{1}{2}\gamma_w(x_3 + B - d_g)(\frac{h-h'}{3})$	$\frac{2}{3}(x_3 + B - d_g)$
10		$U_4 = \gamma_w(x_3 + B - d_g)h'$	$\frac{1}{2}(x_3 + B - d_g)$
11	Silt pressure force	$P_{V_s} = \frac{1}{2} \times 0.925\gamma_w n h_s^2$	$x_3 + B + x_2 - \frac{nh_s}{3}$
12		$EV_1 = \alpha_V W_1$	$x_3 + B + \frac{1}{3}x_2$
13		$EV_2 = \alpha_V W_2$	$x_3 + \frac{B}{2}$
14	Seismic force	$EV_3 = \alpha_V W_3$	$\frac{2}{3}x_3$
15		$EV_4 = \alpha_V P_{V_1}$	$x_3 + B + \frac{2}{3}x_2$
16		$EV_5 = \alpha_V P_{V_2}$	$x_3 + B + \frac{1}{2}x_2$
17		$EV_6 = \alpha_V P_{V'}$	$\frac{mh'}{3}$

Table 2: Computation of horizontal forces acting on the gravity dam.

No.	Category	Horizontal force	Liver arm about toe
1	Water force	$P_H = \frac{1}{2}\gamma_w h^2$	$\frac{h}{3}$
2		$P_{H'} = \frac{1}{2}\gamma_w h'^2$	$\frac{h'}{3}$
3	Silt pressure force	$P_{H_s} = \frac{1}{2} \times 0.36\gamma_w h_s^2$	$\frac{h_s}{3}$
4	Wave pressure force	$P_W = 2\gamma_w h_w^2$	$h + \frac{3}{8}h_w$
5	Seismic forces	$EH_1 = \alpha_H W_1$	$\frac{x_1}{3}$
6		$EH_2 = \alpha_H W_2$	$\frac{H}{2}$
7		$EH_3 = \alpha_H W_3$	$\frac{x_4}{3}$
8		$P_{eH} = 0.726 p_{eH} h,$ $p_{eH} = C_m \alpha_H \gamma_w h,$ and $M_{eH} = 0.299 C_m \alpha_H \gamma_w h^3.$	-

9	$P_{eH'} = 0.726 p_{eH'} h',$ $p_{eH'} = C'_m \alpha_H \gamma_w h',$ *and* $M_{eH'} = 0.299 C'_m \alpha_H \gamma_w h'^3.$	-

Stresses at toe and heel can be calculated as follows:
Principal stress at toe

$$\sigma_{pD} = \sigma_{yD} \sec^2 \phi_D - (p'_H - p_{eH'}) \tan^2 \phi_D, \tag{1}$$

where

$$\sigma_{yD} = \frac{\sum F_V}{B_1}\left(1 + \frac{6e}{B_1}\right),\ p'_H = \gamma_w h'. \tag{2}$$

Principal stress at heel

$$\sigma_{pU} = \sigma_{yU} \sec^2 \phi_U - (p_H + p_{eH}) \tan^2 \phi_U, \tag{3}$$

in which

$$\sigma_{yU} = \frac{\sum F_V}{B_1}\left(1 - \frac{6e}{B_1}\right),\ p_H = \gamma_w h. \tag{4}$$

Shear stress at toe

$$\tau_{xyD} = [\sigma_{yD} - (p'_H - p_{eH'})] \tan \phi_D, \tag{5}$$

Shear stress at heel

$$\tau_{pU} = [\sigma_{yU} - (p_H + p_{eH})] \tan \phi_U. \tag{6}$$

Normal vertical stresses at toe and heel are σ_{yD} and σ_{yU}, respectively. ϕ_D and ϕ_U denote downstream and upstream face slope angles of the dam, respectively. Other parameters are outlined in Tables 2 and 3 and more details can be found in [2, 5].

Table 3: Definitions of the parameters.

Symbol	Name	Symbol	Name
B	Top width of dam	γ_c	Specific weight density of dam material
B_1	Bottom width of dam	γ_w	Specific weight density of water
H	Total height of dam	μ	Coefficient of friction
h	Maximum reservoir level	v_w	Design wind velocity
h'	Tail water level	e	Eccentricity of the resultant force on the dam section
h_s	Silt deposit height	f	Fetch of water on upstream side of dam
$1/m$	Downstream face slope	α_H, α_V	Horizontal and vertical seismic coefficients
$1/n$	Upstream face slope	P_{eH}, $P_{eH'}$	Hydrodynamic pressure force due to earthquake
d_g	Centre of drainage gallery from axis of the dam	$p_{eH}, p_{eH'}$	Hydrodynamic pressure intensity at the base of the dam (head and tail)
M_R	Resisting moment acting about toe	$P_H, P_{H'}$ $P_{V_1}, P_{V_2}, P_{V'}$	Horizontal and vertical components of head and tail water pressure forces
M_O	Overturning moment acting about toe	U_1, U_2, U_3, U_4	Uplift pressure forces
$\sum F_V$	Total vertical force acting on dam	P_{H_s}, P_{V_s}	Horizontal and vertical components of silt pressure force
$\sum F_H$	Total horizontal force acting on dam	P_W	Pressure force due to waves on reservoir
q	Permissible shear stress at foundation	W_1, W_2, W_3	Self weight of dam
σ_c	Permissible compressive strength of concrete	$M_{eH}, M_{eH'}$	Moments due to P_{eH}, $P_{eH'}$

Stability safety factors are computed by the following relationships:

$$FOS = \frac{\sum M_R}{\sum M_O}, \tag{7}$$

$$FSS = \frac{\mu \sum F_V}{\sum F_H}, \tag{8}$$

$$SFF = \frac{\mu \sum F_V + qB_1}{\sum F_H}. \tag{9}$$

FOS is factor of safety against overturning which should be greater than 1.5; *FSS* is the factor of safety against sliding which should also be greater than 1.5; and *SFF* is the shear friction factor which should be greater than 3, Refs. [2, 3, 5, 19].

3. SHAPE OPTIMIZATION PROBLEM FOR A GRAVITY DAM

Gravity dam optimization problem is explained in this section [5]. The cross-sectional area of the dam is considered as an objective function to be minimized. An optimization problem subjected to design constraints can be expressed as follows:

$$\begin{aligned} &Minimize \quad A(\mathbf{X}) \\ &\text{subject to} \quad g_j(\mathbf{X}) \le 0 \\ &\mathbf{X} = [x_1, x_2, ..., x_{nv}] \\ &i = 1,2,...nv \\ &x_i \in \Re^d \end{aligned} \tag{10}$$

$$to\ optimize\ \ Obj(\mathbf{X}) = A(\mathbf{X}) \times f_{penalty}(\mathbf{X}) \tag{11}$$

where **X** is the vector of design variables containing the optimization variables as [n, m, x_1, α_V, α_H]. nv is the number of design variables which is 5 herein, and $\Re^d$ is the domain of the design variables. $Obj(\mathbf{X})$ is the objective function, $A(\mathbf{X})$ is the cross sectional area of the gravity dam, and $f_{\text{penalty}}(\mathbf{X})$ is a penalty function to convert a constrained problem to unconstrained one:

$$A(\mathbf{X}) = 0.5x_1x_2 + BH + 0.5x_3x_4, \tag{12}$$

$$f_{penalty}(\mathbf{X}) = (1 + \kappa_1 \cdot \nu)^{\kappa_2} \ , \ \nu = \sum_{i=1}^{n} \max[0, v_i]. \tag{13}$$

Here, the parameter κ_1 for the penalty function is selected as 1 and the parameter κ_2 is a linearly increasing function ranging from 1.5 to 3. ν indicates the sum of the violated constraints.

Lower bounds and upper bounds of the design variables are described as:

$$\begin{aligned} &0.1 \le n \le 0.2, \\ &0.6 \le m \le 0.9, \\ &0.8h \le x_1 \le 0.95h, \\ &0.05 \le \alpha_V \le 0.2, \\ &0.05 \le \alpha_H \le 0.2. \end{aligned} \tag{14}$$

Moreover, six design constraints are considered as follows:

Stress constraints

$$
\begin{aligned}
g_1(\mathbf{X}) &= \sigma_{pD} - \sigma_c \le 0, \\
g_2(\mathbf{X}) &= \sigma_{pU} - \sigma_c \le 0, \\
g_3(\mathbf{X}) &= \tau_{xyD} - \sigma_c \le 0, \\
g_4(\mathbf{X}) &= \tau_{xyU} - \sigma_c \le 0,
\end{aligned}
\tag{15}
$$

Stability constraints

$$
\begin{aligned}
g_5(\mathbf{X}) &= 1.5 - FOS \le 0, \\
g_6(\mathbf{X}) &= 3 - SFF \le 0.
\end{aligned}
\tag{16}
$$

4. OPTIMIZATION ALGORITHMS

This part briefly explains the concepts behind the algorithms used here as optimizers. The mathematical relations are being addressed to the referred documents [15].

4.1 Charged system search

Charged system search algorithm has been proposed by Kaveh and Talatahari [16] and widely applied to structural optimization such as [11, 12, 15]. This algorithm is established upon Coulomb law from electrostatics and the Newtonian laws of classic mechanics. The CSS is a population based algorithm such that each agent is called a charged particle (CP) that is supposed to be a sphere with constant radius having uniform charge density under the effect of other particle's force field. The value of the resultant force is clarified by using the electrostatics laws and the quality of the movement is determined using Newtonian mechanics laws. Unlike a bad CP, a good CP must induce more force. The main rules of the CSS are as follows:

Rule 1: At each iteration, a pre-determined number of agents are utilized to explore the search space and the magnitude of the charge for each agent or CP, and the separation distance between two charged particles is defined.

Rule 2: The initial positions of the CPs are determined randomly in the search space and the initial velocities of the charged particles are considered as zero.

Rule 3: Electric forces between any two CPs are assumed to be attractive.

Rule 4: All good CPs can attract the bad CPs and only some of the bad agents can attract good agents, considering a probability function.

Rule 5: The value of the resultant electrical force influencing a CP is determined.

Rule 6: The new position and the velocity of each CP are determined.

Rule 7: The CSS uses a memory (CM) which saves the best CP vectors and their related objective function values.

Rule 8: The agents violating the limits of the variables are regenerated using the harmony search scheme for handling approach.

Rule 9: Finally, a terminating criterion leads to stop the process.

4.2 Colliding bodies optimization

Colliding bodies optimization is a meta-heuristic algorithm that was recently developed by Kaveh and Mahdavi [17] and implemented to structural design optimization [13, 15]. In this method, one object collides with other object and they move towards a minimum energy level. The CBO is simple in concept and does not depend on any internal parameter. Each colliding body (CB) has an especial mass defined according to fitness evaluation. In order to select pairs of objects for collision, CBs are sorted according to their mass in a descending order and they are grouped into two equal category: (I) stationary group, (II) moving group. Moving objects collide to stationary objects to make better their positions and push stationary objects towards finer positions. The velocities of the stationary and moving bodies before collision are calculated and the velocity of stationary and moving CBs after the collision are then evaluated. The process is continued until the termination criterion is fulfilled.

4.3 Enhanced colliding bodies optimization

Enhanced colliding bodies optimization was presented [18] to improve the CBO by utilizing a memory called colliding memory (CM) to save a number of historically best CBs. This improvement was inspired from harmony search algorithm [20]. The operation steps of the ECBO are provided as follows:

Level 1: Initialization

Step 1: The initial positions of all CBs are randomly determined in an *m*-dimensional search space. *m* is the number of variables.

Level 2: Search

Step 1: The value of mass for each CB is evaluated.

Step 2: The CM is utilized to save a number of historically best CB vectors and their related masses and objective function values. Solution vectors which are saved in CM are added to the population and the same number of current worst CBs is eliminated. Ultimately, CBs are sorted according to their masses in a descending order.

Step 3: CBs are divided into two equal groups: (I) stationary group, (II) moving group.

Step 4: The velocities of stationary and moving bodies before the collision are evaluated.

Step 5: The velocities of stationary and moving bodies after the collision are calculated.

Step 6: The new position of each CB is computed.

Step 7: A parameter called ***Pro*** within domain of (0, 1) is defined and it is specified whether or not a component of each CB must be changed. For each colliding body ***Pro*** is compared with a random number uniformly distributed within (0, 1). If the random number is less than ***Pro***, one dimension of a CB is selected randomly and its value is regenerated. For pattern protection of CBs, only one dimension is changed.

Level 3: The termination criterion check.

Computer programs for the CBO and ECBO are available in [14].

5. ILLUSTRATIVE EXAMPLE

In this section, three recently developed optimization algorithms consisting of the CSS, CBO and ECBO are utilized for a stability based design optimization of a gravity dam.

Kshirsagar [2] accomplished a case study on the effect of variation of earthquake intensity on stability of gravity dam of Tilari project located at village Dhamane, Taluka Chandgad, District Kolhapur of Maharashtra State. The same data is utilized to optimize its cross-section [5] by differential evolution algorithm. This dam was constructed in 1986 on Tilari river and is located at seismic zone III according to I.S. criteria [3]. Length of the gravity dam is 485 m and maximum flow rate is designed to be 1028 m^3/s. Constant parameters for this structure are listed in Table 4. These parameters are previously defined in Table 3 and Fig. 1.

Table 4: Constant parameters.

Parameter	Assigned value	Parameter	Assigned value
B	4.9 m	x_4	33.35 m
H	38.55 m	h	36.2 m
γ_c	$2.4\gamma_w$	h'	3 m
γ_w	9.81 kN/m^3	h_s	13 m
μ	0.75	d_g	1 m
σ_c	3000 kPa	v_w	80 km/h
q	1200 kPa	f	10 km

Number of particles and iteration for all the employed algorithms are selected as 20 and 100, respectively. Fig 2. compares convergence curves of the three algorithm for the optimization and Table 5 shows optimum values of variables and their corresponding objective function (i.e., cross sectional area) attained by the CSS, CBO and ECBO. As it is visible, the algorithms converge to nearly identical objective function, while the optimized variables are slightly different and Table 6 demonstrates that the stability and stress values are different as well. These values are computed for obtained optimal designs. Obviously, no constraints are violated as it is realized from Table 6. Results illustrate the ability of the applied algorithms for optimal design of the gravity dam, all of them have achieved to stable solution.

Table 5: Optimum designs obtained by algorithms.

Design variables	DEA [5]	CSS	CBO	ECBO
n	0.1	0.100000	0.100000	0.100000
m	0.6	0.60000	0.600000	0.600000
x_1 (m)	28.96	28.960000	28.960000	28.960000
α_V	0.053	0.058953	0.050172	0.050000
α_H	0.064	0.055829	0.051441	0.050000
Cross sectional area (m^2)	564.496	564.49583	564.49583	564.49583

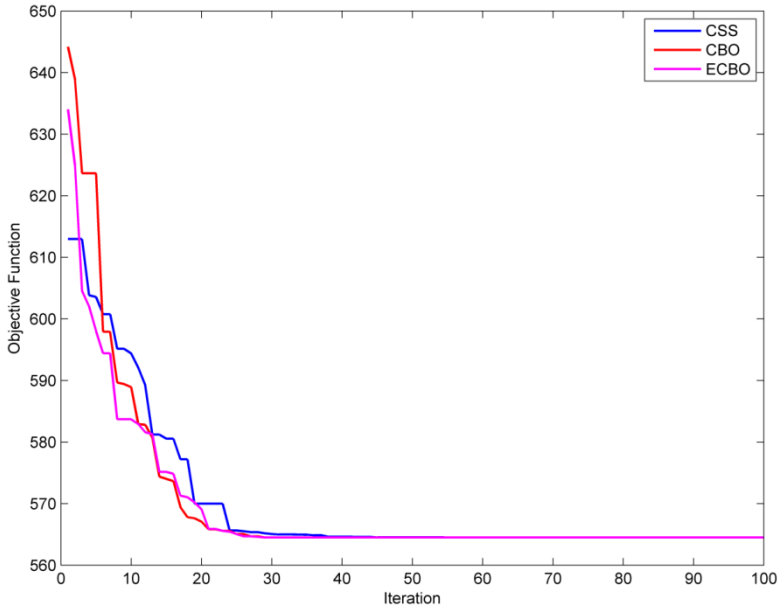

Figure 2. Convergence history comparison for the CSS, CBO and ECBO.

Table 6: Stresses, moments and safety factors for optimum designs.

Parameter	DEA [5]	CSS	CBO	ECBO
σ_{PD}	N.A.	989.20658081646	1001.22347101506	1001.47506651963
σ_{PU}	N.A.	0.0839413353798519	-0.201753078715746	-0.295642226514136
τ_{xyD}	N.A.	424.180173869754	429.422851796678	429.514496093057
τ_{xyU}	N.A.	-36.8953798393777	-36.7866725968693	-36.7509477639856
M_R	N.A.	248852.921459401	248852.920393899	248852.920393899
M_O	N.A.	164249.438613684	160855.972354681	160410.023048892
FOS	N.A.	1.51509145821013	1.54705427937229	1.55135518132835
SFF	N.A.	5.24971388761417	5.32068259515225	5.3406744597817

6. DISCUSSION

Some points are worth to be motioned here. In this paper, *SFF* formulation, Eq. (9), [2, 5] is corrected based on Ref. [19]. However, the obtained value is correct in the case study solved in [2]. The variable x_1 was restricted to lower bound and upper bound of 0.8*H* and 0.95*H* in [5], respectively. 0.8*H* is equal to 30.84m, whereas the optimum value was given as 28.96m in [5] which is less than the pre-defined lower bound. Thus, these values are corrected here to 0.8*h* and 0.95*h*, respectively, as they were typos. In relation with the optimization problem, it seems that all algorithms should have reached to a closely global optimum.

Although a lot of relationships are used, the optimization problem is too simple and it can be expanded by more shape variables or additional constraints. As a sample, factor of safety against sliding, Eq. (8), may be added, requiring more base frictional strength of the dam. Also, selection of α_H and α_V as optimization variables are not very reasonable, because they are usually chosen for a determined seismic zone where the structure is built in that zone, and therefore they must be constant values. It is reasonable to be variables when a study on the effects of seismic coefficient variations on the seismic design optimization is under consideration. Nevertheless, the problem is kept unchanged here for better evaluation and comparison of the algorithms' performances and extension or changing optimization problem is not the purpose of this work.

7. CONCLUDING REMARKS

In this paper shape optimization of a gravity dam is conducted considering stability and stress constraints. Three recently proposed meta-heuristic optimization algorithms are implemented for the dam optimization. These algorithms consist of charged system search (CSS), colliding bodies optimization (CBO) and enhanced colliding bodies optimization (ECBO). Results demonstrated the ability of these algorithms for this type of continuous optimization problem. Status of optimally designed dam for each algorithm is also provided for better assessment of safety factors, overturning and resisting moments, and stress values. It is shown that all the constraints are completely satisfied. This research provides useful formulations for stability analysis of gravity dams and can be extended to other dams.

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