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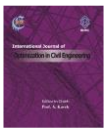

GENERALIZED FLEXIBILITY-BASED MODEL UPDATING APPROACH VIA DEMOCRATIC PARTICLE SWARM OPTIMIZATION ALGORITHM FOR STRUCTURAL DAMAGE PROGNOSIS

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ABSTRACT

This paper presents a new model updating approach for structural damage localization and quantification. Based on the Modal Assurance Criterion (MAC), a new damage-sensitive cost function is introduced by employing the main diagonal and anti-diagonal members of the calculated Generalized Flexibility Matrix (GFM) for the monitored structure and its analytical model. Then, the cost function is solved by Democratic Particle Swarm Optimization (DPSO) algorithm to achieve the optimal solution of the problem lead to damage identification. DPSO is a modified version of standard PSO algorithm which is developed for presenting a fast speed evolutionary optimization strategy. The applicability of the method is demonstrated by studying three numerical examples which consists of a ten-story shear frame, a plane steel truss and a plane steel frame. Several challenges such as the efficiency of the DPSO algorithm in comparison with other evolutionary optimization approaches for solving the inverse problem, impacts of random noise in input data on the reliability of the presented method, and effects of the number of available modal data for damage identification, are studied. The obtained results reveal good, robust and stable performance of the presented method for structural damage identification using only the first several modes' data.

Keywords: damage detection; modal data; generalized flexibility matrix (GFM); modal assurance criterion (MAC); democratic particle swarm optimization (DPSO).

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1. INTRODUCTION

Performance of civil infrastructures, such as high-rise buildings, bridges, tunnels, offshore platforms and other structures may be critically influenced by existence of structural damages or even lead to devastating consequences. Hence, a great deal of attention has been drawn to detect structural damages induced by earthquakes, explosions or hurricanes immediately after the event or monitoring long-term for remedy or repair. Generally, damage is defined as some changes in the physical properties of a structure, such as stiffness matrix. On the other hand, presence of damage causes changes in structural dynamic characteristics, such as mode shape vectors and natural frequencies, and due to the relation between structural physical properties and dynamic characteristics, it seems that damages can be detected by inspecting changes in the dynamic characteristics. This idea has been considered as basic concept behind of vibration-based techniques for damage identification [1].

Various approaches and several response parameters for identifying damage on structures have been proposed [1, 2]. Generally, vibration-based damage detection can be classified as model-based and non-model-based methods. Non model-based methods or index methods (such as [3–6]) rely on signal processing techniques which are developed due to the fact the defects cause usually the reduction in the rigidity of the structure which results in the change of the vibratory characteristics (like eigen frequencies and eigen modes). Although these methods can efficiently localize structural damage, they cannot be useful for damage quantification.

Model-based methods identify damage severity as well as damage location by correlating damaged and analytical models based on the finite element theory. The finite element method can be employed for damage detection by inverse techniques or models updating. The basic concept of finite element model updating is seeking the solution domain to find a good arrangement of unknown damage severities in the analytical model of damaged structure which can generate the same measured modal data from monitored structure. Although there are different approaches which update structural model by a direct methodology for damage estimation [7–10], defining damage detection problem as an inverse model updating problem and solving it by optimization algorithms have received considerable attention in the recent years [11–25]. Begambre and Laier [26] calculated frequency response function of a structure under sinusoidal excitation by employing structural modal data and introduced a new damage-sensitive cost function which was solved by a hybrid Particle Swarm Optimization-Simplex algorithm. Ghodrati Amiri *et al.* [13] employed Pattern Search and Genetic Algorithm for damage identification in different kinds of plates. Meruane and Heylen [27] proposed several modal data-based cost functions and solved them by hybrid real Genetic Algorithm for detecting structural damages. Imperialist Competitive Optimization Algorithm was utilized by Bagheri *et al.* [17] for presenting a new structural damage detection method which was based on free vibration equilibrium of structures. By considering natural frequency changes as a damage-sensitive parameter, Saada *et al.* [19] defined damage detection problem as an inverse problem and solved it by means of Particle Swarm Optimization for identifying damage in beams. Tabrizian *et al.* [20] used Big Bang-Big Crunch algorithm for finding optimal solution of the proposed modal data-based cost function for structural damage detection and estimation. Nouri Shirazi *et al.* [25] identified structural damage by employing an adaptive multi-stage

optimization method based on the modified Particle Swarm algorithm for solving an inverse problem which was based on evaluating the first several natural frequencies of the structure before and after damage. Cha and Buyukozturk [28] utilized a hybrid multi-objective Genetic Algorithm for minor damage localization and quantification in steel structures by means of modal strain energy. Kaveh and Maniat [29] proposed an optimization-based methodology for detecting structural damage and illustrated that the Magnetic Charged System Search algorithm performs better than Particle Swarm Optimization in finding optimal solution. Recently, Zare Hosseinzadeh *et al.* [30] suggested a new method for damage identification in engineering structures using estimated static displacements by flexibility matrix and Cuckoo Optimization Algorithm.

Despite the good performance of the mentioned optimization-based methods, approaches which need smaller amount of data are much preferred. Although it may seems that using more data can boost the accuracy, its disadvantages, such as intensification of noise effects, outnumber its benefits and it seems that methods with utilizing as low as possible input data for damage identification are more reasonable especially in real Structural Health Monitoring (SHM) programs. In addition, researchers try to find simple and fast methods, and estimate damages with low costs.

In this paper a new methodology for localizing and also estimating severity of structural damage is proposed. By considering the main diagonal and anti-diagonal members of the calculated Generalized Flexibility Matrix (GFM) in the damaged and undamaged states (which can be calculated using only the first several lower modes' data), a new damage-sensitive cost function is proposed that is based on geometrical correlation measurement via Modal Assurance Criterion (MAC). Finally, Democratic Particle Swarm Optimization (DPSO) algorithm, a modified version of the standard Particle Swarm Optimization (PSO), is utilized to minimize the objective function and report structural damages. To validate the applicability of the presented method for structural damage identification, three numerical examples of engineering structures are studied and different challenges such as the performance of the suggested method in the presence of random noises and the effects of the number of available modal data as well as the robustness of the DPSO algorithm in comparison with other evolutionary optimization methods for solving inverse problems, are investigated.

The paper is organized as follows. The overview of the DPSO algorithm is presented in Section 2. Then, the damage identification method is described in Section 3. It is followed by Section 4 which introduces the numerical examples and presents the obtained results. Finally, the paper ends with some conclusion remarks.

2. DEMOCRATIC PARTICLE SWARM OPTIMIZATION (DPSO)

Particle Swarm Optimization (PSO) is a famous population-based stochastic optimization technique that optimizes a problem by iterative tries to improve a potential solution with a given measure of quality developed by Kennedy and Eberhart [31], inspired by social behavior of bird flocking or fish schooling. Each particle moves with a velocity around the multidimensional search space which is continuously updated by the particle's own experience and the experience of the particle's neighbors or the experience of the entire

particles. Advantages of PSO caused to draw a great deal of attention in different fields of science. Nevertheless, PSO does show some disadvantages: sometimes it can be trapped in local optima easily, and the convergence rate decreased considerably in the later iteration of evolution; when reaching a near optimal solution, the algorithm stops optimizing which cause to decrease the accuracy of the algorithm.

Democratic Particle Swarm Optimization (DPSO) has been introduced by Kaveh and Zolghadr [32] as an improved version of the standard PSO to tackle original PSO own set of drawbacks mentioned above. In fact DPSO is an effort to provide a better tactic for searching the solution domain by taking the experiences of all kinds of particles either qualified particles or bad particles and this strategy can avoid the premature convergence. The improvement is obtained by adding a new term to the velocity vector. The velocity vector of DPSO is expressed as:

$$v_{i,j}^{k+1} = \chi[wv_{i,j}^{k} + c_1 r_1 (xlbest_{i,j}^{k} - x_{i,j}^{k}) + c_2 r_2 (xgbest_{j}^{k} - x_{i,j}^{k}) + c_3 r_3 d_{i,j}^{k}] \tag{1}$$

where, w indicates the inertia weight for the previous iteration's velocity and χ is a parameter for preventing divergence behavior. These parameters can be calculated and selected based on the stated formulation in Kaveh and Zolghadr [32]. $v_{i,j}^{k}$ is the velocity of variable j of the i-th particle, $x_{i,j}^{k}$ is the current value of the j-th variable of the i-th particle, $xlbest_{i,j}^{k}$ is the best value of the j-th variable which can be found by i-th particle and $xgbest_{j}^{k}$ is the best value of the variable j experienced by the whole particles so far. r_1, r_2 and r_3 are three random constants which are distributed uniformly in the range of (0,1). c_1 and c_2 are parameters for demonstrating rate of particle's confidence in itself and in the swarm, respectively. c_3 is a parameter which control the weight of the democratic vector. $d_{i,j}^{k}$ stands for j-th variable of the vector **D** for the i-th particle. The vector **D** denotes the democratic influence of the other particles of the swarm on the movement of the i-th particle and is considered as:

$$D_i = \sum_{k=1}^{n} Q_{ik} (X_k - X_i) \tag{2}$$

where Q_{ik} is the weight of the k-th particle in the democratic movement of the i-th particle and is calculated as:

$$Q_{ik} = \frac{E_{ik} \dfrac{f_{best}}{f(k)}}{\sum_{j=1}^{n} E_{ij} \dfrac{f_{best}}{f(j)}} \tag{3}$$

in which f is cost function value. In addition, f_{best} is the value of cost function for the best particle in current iteration, X is the particle's position vector, and E is the eligibility parameter. For minimization problems E is defined as:

$$E_{ik} = \begin{cases} 1 & \dfrac{f(k)-f(i)}{f_{worst}-f_{best}} > rand \cup f(k) < f(i) \\ 0 & else \end{cases} \tag{4}$$

where f_{worst} is the value of cost function for worst particle, and f_{best} is the value of cost function for best particles in the current iteration. After calculating velocity by Eq. (1), the new positions of the particles in DPSO algorithm are defined similar to the standard PSO as below:

$$x_{i,j}^{k+1} = x_{i,j}^{k} + v_{i,j}^{k+1} \tag{5}$$

in which the time interval is equal to 1.0 and thus the velocity vector can be added to the position vector. It is clear that the information produced by all of members of the swarm is utilized by DPSO with the purpose of determine the new position of each particle. Actually, according to the above mentioned procedure for calculating particles' velocity, the new position of the particle is defined with consideration all of the better particles and also some of the worse particles. DPSO can be assumed as a perfect searching approach in which all candidate points of the solution domain can be scanned approximately for finding the global extremum and associated variables.

3. PROPOSED METHOD

This section is devoted to explain the details of the suggested method for structural damage prognosis. For a structure with N degrees of freedom (DOFs) and N_e elements, the free vibration equation can be presented as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0} \tag{6}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the global structural mass and stiffness matrices, respectively. Also, $\ddot{\mathbf{x}}$ and $\mathbf{x}$ are the acceleration and displacement vectors, respectively. Structural modal data can be extracted by solving free vibration equilibrium which is introduced as below:

$$\mathbf{K}\boldsymbol{\varphi}_i = \omega_i^2 \mathbf{M}\boldsymbol{\varphi}_i \ , \ i = 1, 2, ..., N \tag{7}$$

Where ω_i and $\boldsymbol{\varphi}_i$ are the i-th natural frequency and related mass-normalized mode shape vector, respectively. The relation between modal data and stiffness matrix can be expressed as:

$$\mathbf{K} = \boldsymbol{\Phi}^{-T} \boldsymbol{\Lambda} \boldsymbol{\Phi}^{-1} \tag{8}$$

in which $\boldsymbol{\Phi}$ is a matrix which consists of structural eigenvectors and $\boldsymbol{\Lambda}$ is a diagonal matrix that is defined as below:

$$\mathbf{\Lambda} = \begin{bmatrix} \omega_1^2 & 0 & ... & 0 \\ 0 & \omega_2^2 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & \omega_N^2 \end{bmatrix} \tag{9}$$

By inspecting Eq. (8) it is obvious that all modes' data should be accessible for calculating the global stiffness matrix via modal data. Although this relation is correct mathematically, from practical viewpoint, estimating stiffness matrix employing only the first several modes' data is so respected. An acceptable approach for this purpose can be stated by considering the flexibility matrix. Flexibility matrix (**F**) is the inverse of stiffness matrix and can be written as below:

$$\mathbf{F} = \mathbf{\Phi}\mathbf{\Lambda}^{-1}\mathbf{\Phi}^T \tag{10}$$

Based on Eq. (10), the flexibility matrix depends inversely on ω_i^2. Therefore, by increasing the number of utilized modes, the impacts of the natural frequencies on calculating flexibility matrix decrease, extremely. It means that an acceptable estimation for inverse of global stiffness matrix can be achieved by employing only the first '*m*' modes' data. Li *et al.* [33] suggested Generalized Flexibility Matrix (GFM) as an effort to strongly decrease the effects of higher modes' data on calculating the flexibility matrix. *l*-th order of GFM by employing the first '*m*' modes' data is defined by multiplying of $(\mathbf{MF}_m)^l$ in the flexibility matrix as below:

$$\mathbf{F}_m^{g(l)} = \mathbf{F}_m (\mathbf{MF}_m)^l \tag{11}$$

Subscript '*m*' denotes number of utilized modes for constructing matrices. Using Eq. (10) and doing some mathematical simplification, the GFM can be expressed as:

$$\mathbf{F}_m^{g(l)} = \mathbf{\Phi}_m \mathbf{\Lambda}_m^{-1-l} \mathbf{\Phi}_m^T \ , \ l = 0,1,2,... \tag{12}$$

This paper uses the first-order of the GFM (*l*=1):

$$\mathbf{F}_m^{g(1)} = \mathbf{\Phi}_m \mathbf{\Lambda}_m^{-2} \mathbf{\Phi}_m^T \tag{13}$$

Therefore, it can be concluded that the GFM decreases the effects of the natural frequencies of higher modes, strongly and a suitable estimation of the flexibility matrix can be available by utilizing only the first several lower modes' data.

In this paper we use GFM for formulating a new damage-sensitive cost function, by measuring amount of geometrical correlation between vectors. If the calculated GFM based on the first '*m*' modes' data of the monitored (or damaged) structure is denoted by $\mathbf{F}_m^{g(1)}$, the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ are defined as the main diagonal and anti-diagonal members of $\mathbf{F}_m^{g(1)}$ as below:

$$\mathbf{a}_1 = \left\{ \mathbf{F}_m^{g(1)}(1,1) \quad \mathbf{F}_m^{g(1)}(2,2) \quad ... \quad \mathbf{F}_m^{g(1)}(N,N) \right\}^T \tag{14}$$

$$\mathbf{a}_2 = \left\{ \mathbf{F}_m^{g(1)}(1,N) \quad \mathbf{F}_m^{g(1)}(2,N-1) \quad ... \quad \mathbf{F}_m^{g(1)}(N,1) \right\}^T \tag{15}$$

In addition, if the GFM employing the first '*m*' modes' data for undamaged structure is shown by $\mathbf{F}_m^{g(1),u}$, the GFM's alterations between damaged and undamaged states can be calculated as:

$$\Delta\mathbf{F} = \mathbf{F}_m^{g(1)} - \mathbf{F}_m^{g(1),u} \tag{16}$$

Hence, the main diagonal and anti-diagonal members of this matrix are denoted as followings:

$$\mathbf{a}_3 = \left\{ \Delta\mathbf{F}(1,1) \quad \Delta\mathbf{F}(2,2) \quad ... \quad \Delta\mathbf{F}(N,N) \right\}^T \tag{17}$$

$$\mathbf{a}_4 = \left\{ \Delta\mathbf{F}(1,N) \quad \Delta\mathbf{F}(2,N-1) \quad ... \quad \Delta\mathbf{F}(N,1) \right\}^T \tag{18}$$

In the analytical model of damaged structure (with unknown damage severity), damage is defined as some deterioration in the stiffness matrix of the damaged structure. So, the stiffness matrix of the *i*-th element in the damaged state ($\mathbf{k}_i^d$), can be written as:

$$\mathbf{k}_i^d = (1-d_i)\mathbf{k}_i^u \tag{19}$$

where $\mathbf{k}_i^u$ and d_i are the stiffness matrix of the *i*-th element in the undamaged state and unknown damage ratio for the *i*-th element, respectively. It is worth noting that d_i will be zero and 1.0 for undamaged and fully damaged elements, respectively. Overall, the global stiffness matrix of the analytical model of damaged structure with unknown damage severities can be expressed as:

$$\mathbf{K}^d = \bigcup_{i=1}^{N_e} \mathbf{k}_i^d \tag{20}$$

The GFM of the analytical model of damaged structure using the first '*m*' modes' data ($\mathbf{F}_m^{g(1)}|_d$) can be calculated by employing Eq. (13). Then, the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$ can be constructed for analytical state (with unknown damages) using Eqs. (14), (15), (17) and (18), respectively. For analytical model, these vectors are referred by adding a superscript '*d*' (i.e.: $\mathbf{a}_1^d$, $\mathbf{a}_2^d$, $\mathbf{a}_3^d$ and $\mathbf{a}_4^d$).

The proposed cost function is aimed at measuring amount of correlation between vectors ($\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$) and ($\mathbf{a}_1^d$, $\mathbf{a}_2^d$, $\mathbf{a}_3^d$ and $\mathbf{a}_4^d$). In this paper, to measure amount of correlation between two vectors, the MAC parameter is employed. Generally, MAC can be interpreted as a criterion for measuring amount of geometrical correlation between two vectors. For two given vectors of **x** and **y**, MAC is defined as [34]:

$$MAC\left(\mathbf{x},\mathbf{y}\right)=\frac{\left|\mathbf{x}^{T}.\mathbf{y}\right|^{2}}{\left(\mathbf{x}^{T}.\mathbf{x}\right).\left(\mathbf{y}^{T}.\mathbf{y}\right)} \tag{21}$$

The vectors are in complete accordance if MAC is equal to 1.0. For introducing the suggested cost function, first we define:

$$MAC_{i}\left(\mathbf{a}_{i},\mathbf{a}_{i}^{d}\right)=\frac{\left|\left(\mathbf{a}_{i}\right)^{T}.\left(\mathbf{a}_{i}^{d}\right)\right|^{2}}{\left(\left(\mathbf{a}_{i}\right)^{T}.\left(\mathbf{a}_{i}\right)\right).\left(\left(\mathbf{a}_{i}^{d}\right)^{T}.\left(\mathbf{a}_{i}^{d}\right)\right)}\quad,\quad i=1,2,3,4 \tag{22}$$

Then, the cost function is defined as below:

$$f\left(d_{1},d_{2},...,d_{N_{e}}\right)=\sqrt{\left(\frac{1-e_{1}}{e_{1}}\right)^{2}+\left(\frac{1-e_{2}}{e_{2}}\right)^{2}}\quad,\quad 0\le d_{i}\le 1.0 \tag{23}$$

where:

$$e_{1}=MAC_{1}\times MAC_{3} \tag{24}$$

$$e_{2}=MAC_{2}\times MAC_{4} \tag{25}$$

Finally, the DPSO algorithm is employed for solving the presented optimization problem. DPSO is a modified version of standard PSO algorithm propose by Kaveh and Zolghadr [31] and described in Section 2.

4. NUMERICAL STUDIES

In this section the applicability of the presented method is demonstrated by studying three numerical examples of structures under different damage patterns. Moreover, not only is the efficiently of the DPSO in solving inverse problems compared with other evolutionary optimizations, but also, the robustness of the suggested damage detection method in practical cases is investigated by considering some of the important challenges which are available in the real SHM programs.

4.1 A ten-story shear frame

In the first example, the presented method is applied for damage detection in a ten-story shear frame structure. Table 1 describes the physical properties of this structure. In this example studied two damage patterns are summarized in Table 2. In real SHM programs, the input data are contaminated with different levels of random noises. So, for an actual judgement about the feasibility of a damage detection method in real conditions, it is

essential to investigate the case with polluted input data by noise, in addition to consideration of an ideal case (i.e. free noise state). In this paper, the natural frequencies are contaminated by random noises via the presented strategy by Eq. (26) [10]:

$$\omega_i^n = \omega_i (1 + \sigma\kappa_i) \tag{26}$$

where ω_i^n and ω_i are the i-th natural frequency with and without noise, respectively. σ is the noise level, and κ_i is a random value between [-1 1] which is generated by MATLAB software. In this example, 5% random noise is considered in the natural frequencies.

Table 1: Physical properties of ten-story shear frame

Story No.	Mass (ton)	Stiffness (MN/m)
1~3	80	7.5
4~7	55	7.5
8~10	30	5.0

Table 2: Simulated damage patterns in the ten-story shear frame

Damage Pattern I		Damage Pattern II	
Story	Damage (%)	Story	Damage (%)
4	10	2	5
		5	15
		9	10

To investigate the impacts of the number of utilized modal data for calculating GFM (m) on the performance of the suggested method, two different cases are considered using one and three first modes: m=1, and m=3. The presented method is applied for damage identification in the simulated damage scenarios. The optimization parameters are selected as follows: number of particles=100, number of iterations=1000, c_1=2, c_2=2, and c_3=4. There is not any regular strategy for choosing these parameters and they are selected by trial and error approach.

The obtained results are shown in Fig. 1. From this figure it is obvious that the method can efficiently detect structural damages either the ideal input data are fed or the noisy one. In addition, it can be concluded that by increasing number of utilized modal data for constructing GFM in noisy state, some of the healthy stories are seldom reported as a damaged story, but their damage severities are very small and negligible, so this issue cannot have an influence on the correct judgment about situation of stories. Therefore, the presented method can be considered as a viable method for damage identification.

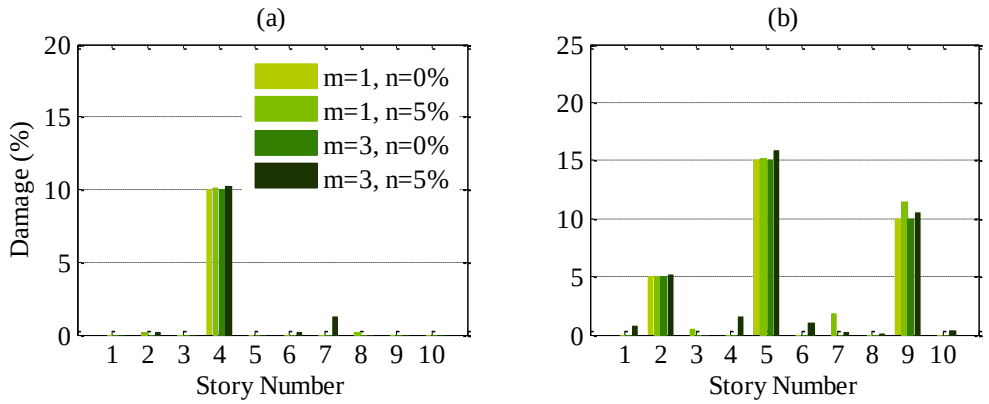

Figure 1. Damage detection results of the ten-story shear frame for (a) damage pattern I and (b) damage pattern II. '*m*' denotes the number of modal data utilized for damage identification and '*n*' shows noise level

Moreover, to demonstrate the applicability of the DPSO algorithm, other evolutionary optimization algorithms are also applied for optimizing the cost function. In this regards, both of the simulated damage patterns, using the first three modes' data (with 5% noise), are assumed again and the suggested cost function is resolved by applying standard PSO algorithm and Genetic Algorithm (GA). For both PSO and GA, the maximum number of iterations is selected equal to 2000. It is worth noting that for precisely selection of parameters of optimization algorithms, at first, a typical scenario without noise is solved by employing PSO and/or GA, then, a trial and error procedure was followed for finding appropriate parameters. Finally, the optimization parameters were selected when the simulated damages got identified with high level of accuracy. Fig. 2 shows the obtained results for the simulated damage patterns. Although the PSO algorithm is able to find damaged stories, there are differences between the predicted damage severities and simulated ones. These differences are even more considerable for the second damage pattern with multiple deteriorations. Premature convergence of the standard PSO because of the complexity of solution domain can be considered as a good reason for this occurrence. However, when the GA is used for damage identification, it is possible that some false-positive and -negative results can be obtained. For instance, in the second damage pattern which consists of damages with small and moderate severities, the small damage in the second element cannot be distinguishably detected. Actually, the GA is arrested by local extremums in searching complex solution domain and therefore, it cannot convergence to global extremums.

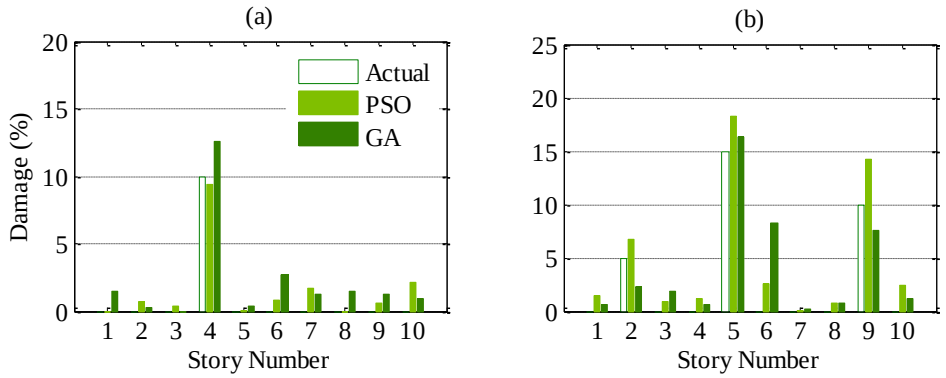

Figure 2. Damage detection results of the ten-story shear frame using PSO and GA for (a) damage pattern I and (b) damage pattern II, when the first three modes' data with 5% noise are utilized

For clearly evaluation the performance of the mentioned optimization algorithms, the convergence curves for DPSO, PSO and GA in solving above addressed scenarios are shown in Fig. 3. As it can be seen, despite using noisy input data, the DPSO algorithm converges to the minimum cost after ~200 iterations, in both damage patterns. Therefore, not only is it approved the presented claims about PSO and GA, but also the DPSO algorithm is introduced as a fast speed optimization procedure in searching complex solution domains. It should be noted that although by increasing number of iterations the performance of the PSO and/or GA may improve; overall, the DPSO shows a fast speed convergence to the global extremum. Therefore, it can be concluded that the DPSO performs better then PSO and GA in finding optimal solution for the presented inverse problem.

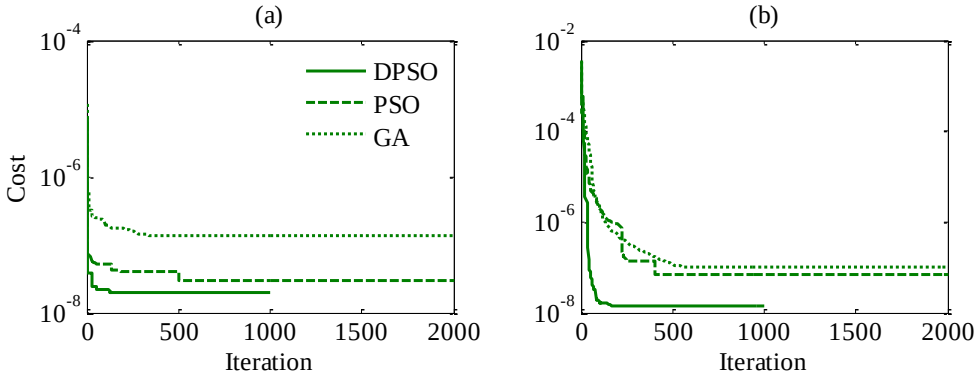

Figure 3. Convergence curves for the DPSO, PSO and GA in solving (a) damage pattern I and (b) damage pattern II of the ten-story shear frame, when the first three modes' data with 5% noise are used

4.2 A planar steel truss

The second example is devoted for damage localization and quantification in a plane steel truss. As shown in Fig. 4, this truss consists of 29 elements and each free node has two degrees of freedom (DOFs). The material properties of this truss are as below: modules of

elasticity E=200 GPa, mass density ρ=7850 kg/m^3, the mass per unit length and cross sectional area for vertical members are m=39.25 kg/m and A=0.005 m^2, and those for bottom horizontal members are m=3000 kg/m and A=0.010 m^2, and those for top horizontal members are m=78.50 kg/m and A=0.010 m^2, and those for the diagonal members are m=62.80 kg/m and A=0.008 m^2, respectively.

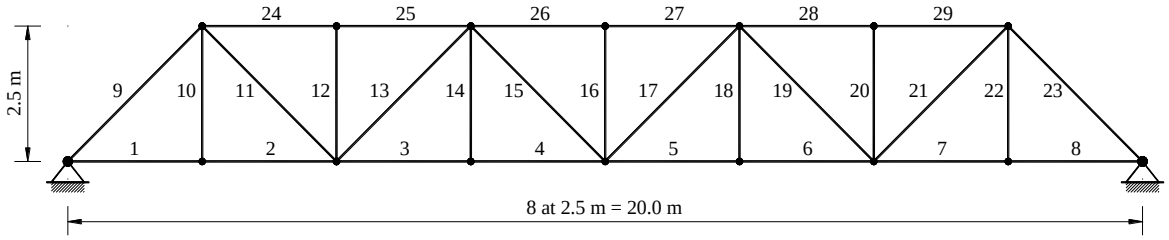

Figure 4. Finite element model of plane steel truss

Table 3 describes simulated damage patterns. Although the first damage case consists of a moderate damage case, the second and third scenarios simulate multiple damage patterns with moderate and severe deteriorations. Similar to the previous example, it is assumed that just the first mode's data and then three first modes' data are available for calculating GFM. In addition, for investigating the noise effects, the input data are contaminated by two levels of random noises (i.e. 3% and 5%). The parameters of the DPSO are selected similar to the previous example. The obtained damage detection results are shown in Figs. 5–7 for the studied three damage patterns. As it is obvious, the suggested cost function is sufficiently sensitive to damage occurrence and is able to localize and quantify damages accurately in the ideal state (i.e. free noise state) as well as noisy state. Therefore, such conclusion can be drawn that the method is a powerful and reliable approach for damage prognosis in real SHM programs.

Table 3: Simulated damage patterns in the plane steel truss

Damage Pattern I		Damage Pattern II		Damage Pattern III	
Element	Damage (%)	Element	Damage (%)	Element	Damage (%)
15	10	11	15	3	30
		22	20	9	10
				25	25

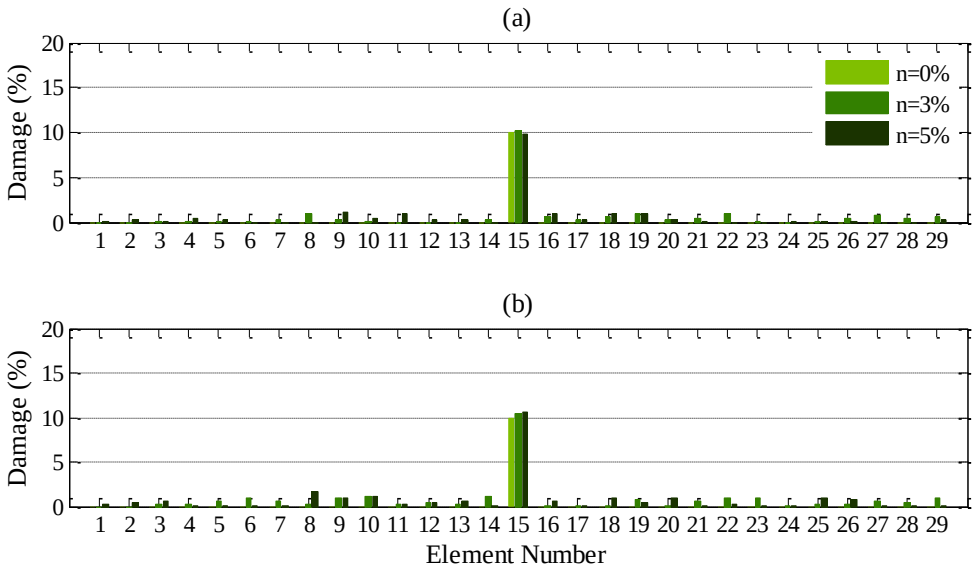

Figure 5. Damage detection results for the first damage pattern of the plane steel truss using (a) the first one mode's data and (b) the first three modes' data. '*n*' shows noise level

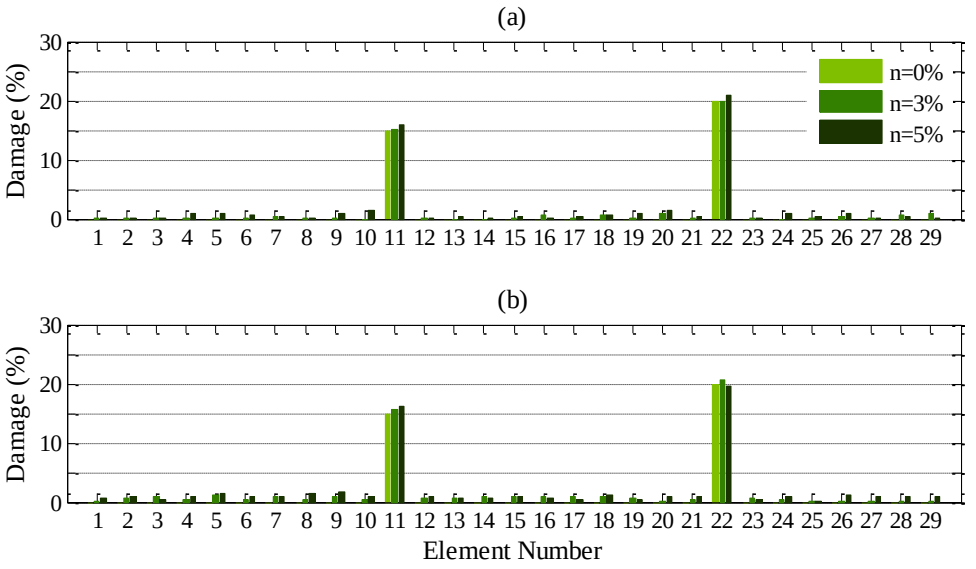

Figure 6. Damage detection results for the second damage pattern of the plane steel truss using (a) the first one mode's data and (b) the first three modes' data. '*n*' shows noise level

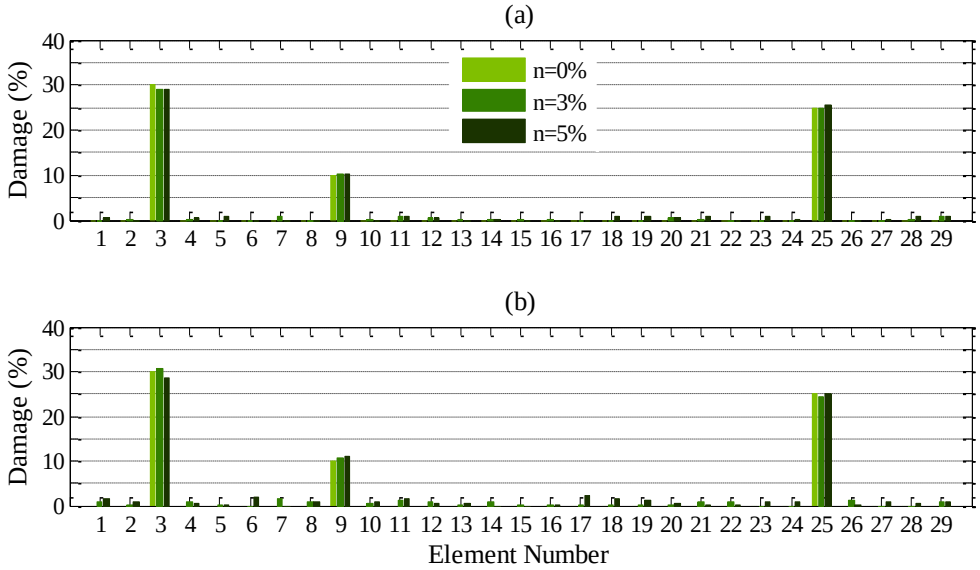

Figure 7. Damage detection results for the third damage pattern of the plane steel truss using (a) the first one mode's data and (b) the first three modes' data. '*n*' shows noise level

This section ends with some studies about evaluating the robustness and stability of the DPSO algorithm in comparison with standard PSO and GA when a unique inverse problem is solved for several times. Although the evolutionary optimization approaches are independent of the initial guesses of the optimal solution or initial population, in the complex problems, it is possible that the method cannot correctly find an appropriate path to reach the optimal solution. Therefore, if an optimization algorithm can reach to an approximately unique solution in different runs of a unique problem, it can be considered as a stable optimization approach. This issue is considered here by resolving simulated damage patterns I and III in the truss (by utilizing the first three modes' data with 5% noise) for 10 times with DPSO, PSO and GA. The optimization parameters are selected similar to the pervious example for all optimization algorithms. For saving space, only the obtained damage severities in the damaged elements are shown in Figs. 8 and 9 for the first and third damage patterns, respectively. Based on these figures, PSO and GA cannot reach to a unique solution in all runs. However, DPSO algorithm shows a good performance and not only it can reach to an approximately unique optimal solution in all runs, but also, it shows an acceptable level of accuracy in estimating severity of damages and it means that the DPSO algorithm is a stable and efficient strategy in searching complex solution domain for finding optimal solution.

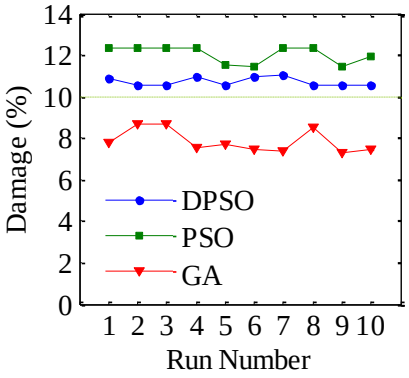

Figure 8. Estimated damage severities in 10 runs for the 15th element of the plane steel truss in the first damage pattern by employing DPSO, PSO and GA, using the first three modes' data with 5% noise

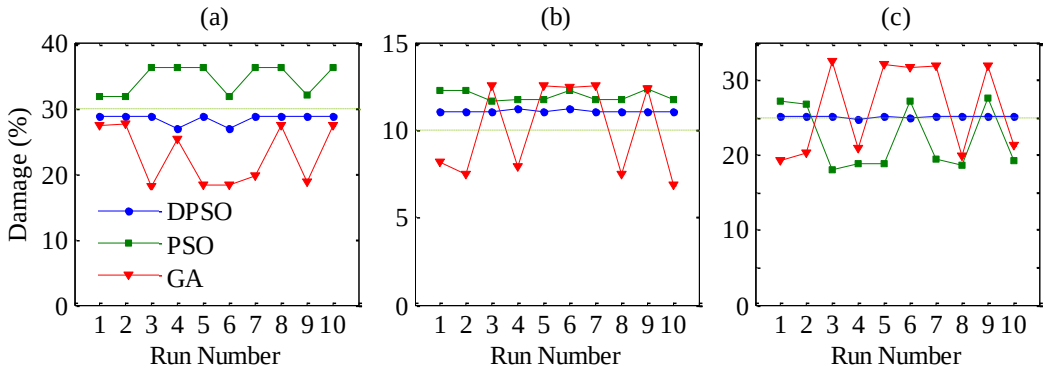

Figure 9. Estimated damage severities in 10 runs in the third damage pattern of the plane steel truss by employing DPSO, PSO and GA, using the first three modes' data with 5% noise for (a) 3rd element, (b) 9th element and (c) 25th element

4.3 A plane steel frame

The last example is concentrated on damage identification in a plane steel frame. The finite element model of this frame is shown in Fig. 10. Each free node of this structure has three DOFs (two translational DOFs in the horizontal and vertical directions and one rotational DOF). For all elements, modules of elasticity and mass density are considered equal to those which were introduced in the previous example. In addition, the mass per unit length, moment of inertia and cross sectional area are equal to 117.7 kg/m, 3.30×10^{-4} m^4 and 0.0150 m^2 for columns, and 1250 kg/m, 3.69×10^{-4} m^4 and 0.0152 m^2 for beams, respectively.

In this example three damage patterns are considered for investigating the performance of the presented method in damage prognosis that are explained in Table 4. The first damage pattern consists of damages with minor and moderate severities. However, the second and third patterns are devoted for multiple damage cases with moderate and severe

deteriorations. Similar to the previous example, the problem is solved by employing only the first mode's data and then the first three modes' data in the free noise state as well as noisy states (3% and 5% noises in input data). In the following, the presented model updating procedure is applied for damage identification in the simulated patterns. The optimization parameters are selected similar to the previous examples. Damage detection results are shown in Figs. 11–13. In the noisy states, although some differences can be seen between simulated and obtained damages which are justifiable because of using noisy input data; overall, the method is able to localize and quantify damages with high level of accuracy. Therefore, we found the presented method as a powerful method for structural damage identification and quantification.

Table 4: Simulated damage patterns in the plane steel frame

Damage Pattern I		Damage Pattern II		Damage Pattern III	
Element No.	Damage (%)	Element No.	Damage (%)	Element No.	Damage (%)
5	10	3	15	4	25
18	5	12	10	7	20
		19	20	14	15
				21	30

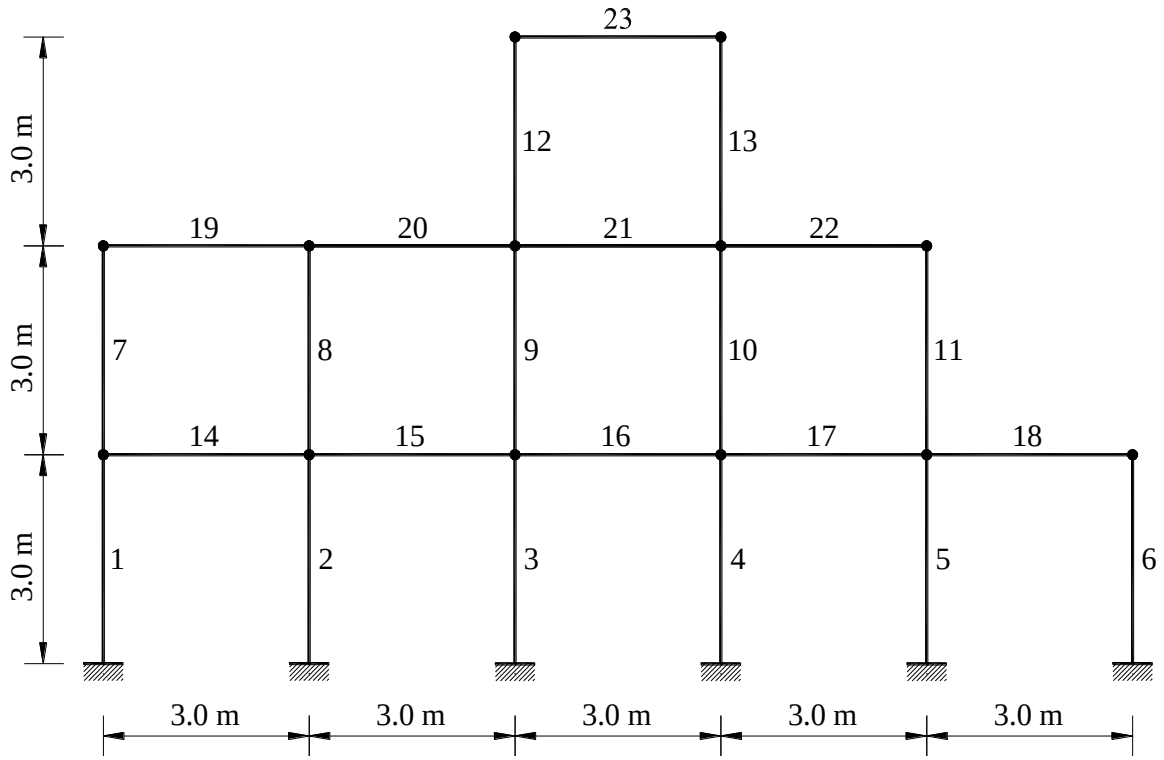

Figure 10. Finite element model of plane steel frame

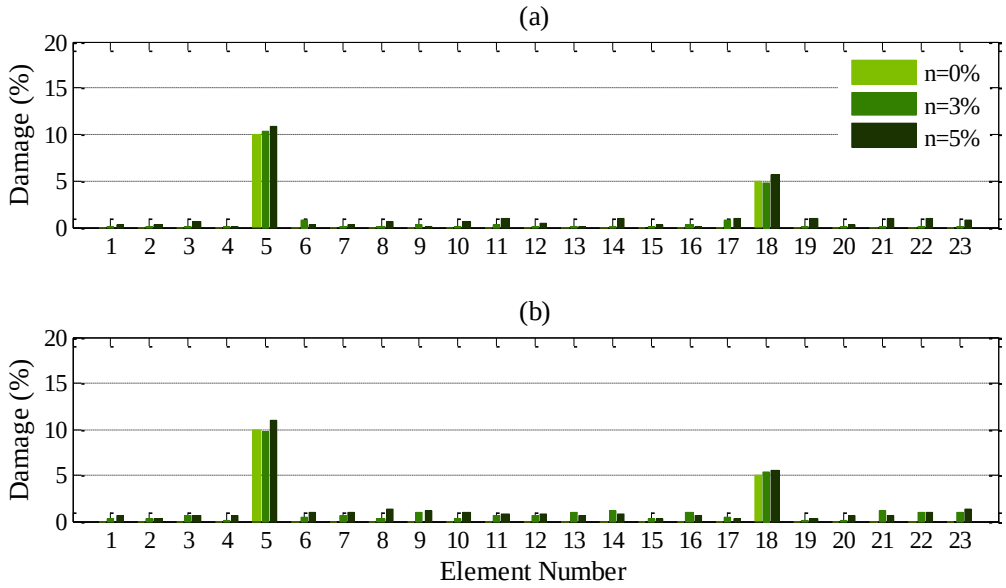

Figure 11. Damage detection results for the first damage pattern of the plane steel frame using (a) the first one mode's data and (b) the first three modes' data. '*n*' shows noise level

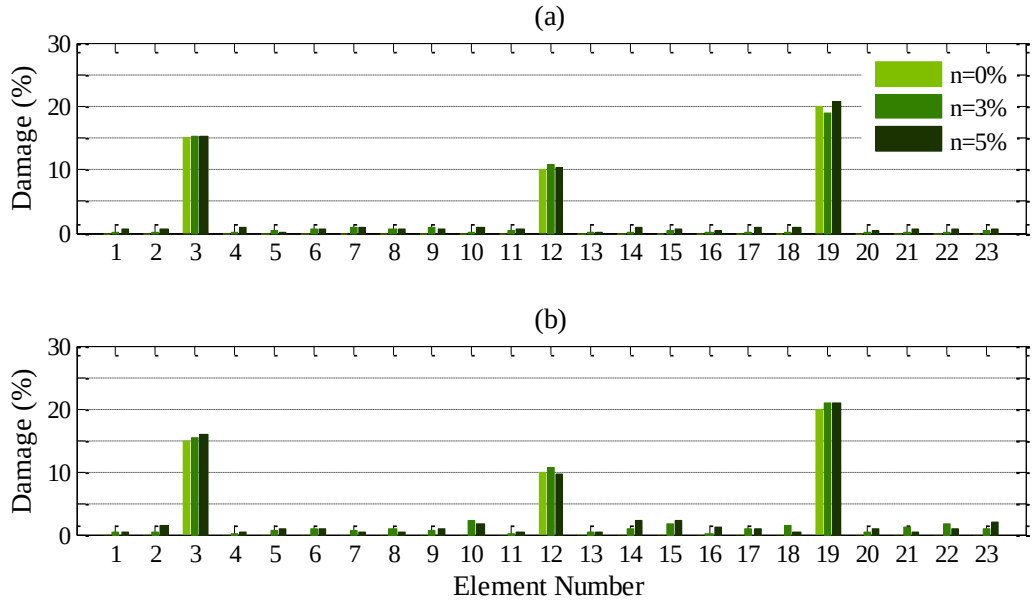

Figure 12. Damage detection results for the second damage pattern of the plane steel frame using (a) the first one mode's data and (b) the first three modes' data. '*n*' shows noise level

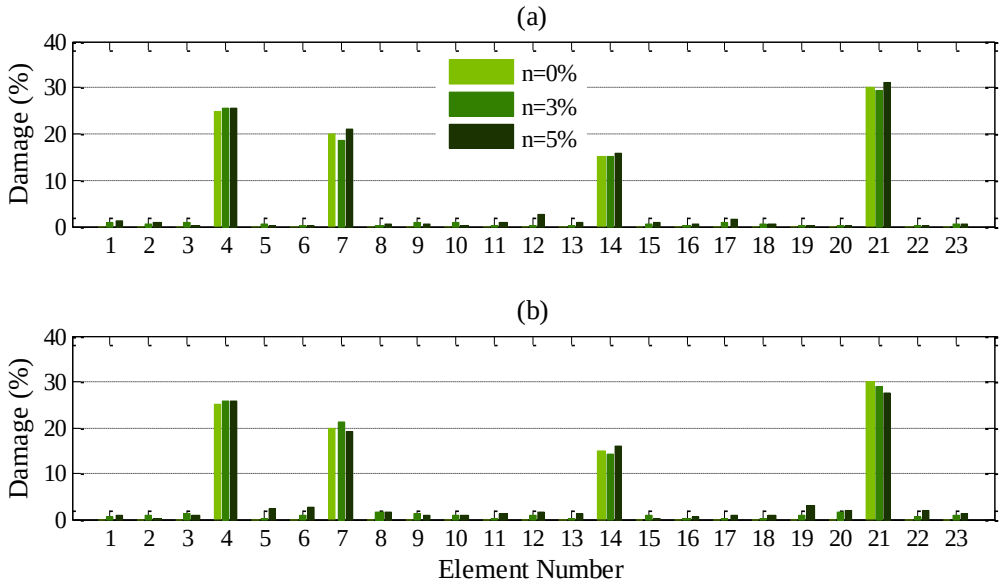

Figure 13. Damage detection results for the third damage pattern of the plane steel frame using (a) the first one mode's data and (b) the first three modes' data. '*n*' shows noise level

5. CONCLUSIONS

This paper introduced an effective model updating-based approach for structural damage identification by employing main diagonal and anti-diagonal members of the Generalized Flexibility Matrix (GFM), which can be estimated using only the first several lower modes' data. The proposed cost function was solved by Democratic Particle Swarm Optimization (DPSO). DPSO is an effort for overcoming to the drawbacks of the standard PSO algorithm by major concentration on preventing from premature convergence to the local extremums [32]. The applicability of the method was demonstrated by studying different damage patterns on three numerical examples. Moreover, the stability of the DPSO algorithm is evaluated by different comparative studies with other evolutionary optimization algorithms, namely, PSO and GA. Some other studies were carried out on evaluating the applicability of the method for damage identification with noisy input data. Results introduced the DPSO algorithm as a fast speed optimization approach which is stable in searching complex solution domain for finding optimal solution. Moreover, the good and acceptable performance of the method for damage detection in real SHM programs was derived out.

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