RELIABILITY–BASED DESIGN OPTIMIZATION OF CONCRETE GRAVITY DAMS USING SUBSET SIMULATION

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ABSTRACT

The paper deals with the reliability–based design optimization (RBDO) of concrete gravity dams subjected to earthquake load using subset simulation. The optimization problem is formulated such that the optimal shape of concrete gravity dam described by a number of variables is found by minimizing the total cost of concrete gravity dam for the given target reliability. In order to achieve this purpose, a framework is presented whereby subset simulation is integrated with a hybrid optimization method to solve the RBDO approach of concrete gravity dam. Subset simulation with Markov Chain Monte Carlo (MCMC) sampling is utilized to estimate accurately the failure probability of dams with a minimum number of samples. In this study, the concrete gravity dam is treated as a two–dimensional structure involving the material nonlinearity effects and dam–reservoir–foundation interaction. An efficient metamodel in conjunction with subset simulation–MCMC is provided to reduce the computational cost of dynamic analysis of dam–reservoir–foundation system. The results demonstrate that the RBDO approach is more appropriate than the deterministic optimum approach for the optimal shape design of concrete gravity dams.

1. INTRODUCTION

In the deterministic optimum design, it is usually assumed that there is no uncertainty in engineering system and finds the minimum cost of engineering system. The deterministic optimum design can lead to a unreliable design and cannot obtaine a balance between cost and safety [1–3]. Hence, a otpimal design procedure must reasonably account for the

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existence of inherent randomness in physical quantities such as element dimensions, material properties and external loads. To achieve this purpose, the reliability–based design optimization (RBDO) has been proposed.

Concrete gravity dams have been widely utilized in dam construction and distinguished as critical structures because of the high cost of their construction, their size and their interactions with the reservoir and foundation. The safety of a dam highly depends on its seismic responses and its capacity which are inherently uncertain. The seismic responses involve the material properties of dam body and its foundation, the water high of dam reservoir and the characteristics associated with ground motions. Hence, the reliability concept should be considered to take into account a number of the possible uncertainties. In addition, the effects of dam–reservoir–foundation interaction and the nonlinear behavior of dam play important roles in the design of new dams and the safety evaluation of existing dams subjected to earthquake loads. In dam engineering, the design of the proper shape for a dam has been distinguished as great challenging problem [4]. Hence, first, several alternative schemes with various patterns should be selected and modified in order to obtain a number of feasible shapes. Then, the final shape of dam is selected from among various patterns, which considers the economy and safety of design, structural considerations, etc.

For many years, the optimal shape design of arch dams as an interesting area of research has received great deal of attentions by numerous researchers [4–10]. In recent years, few attempts have been made for the shape optimization of concrete gravity dams. Salajegheh et al. [11] proposed a hybrid of gravitational search algorithm (GSA) and particle swarm optimization (PSO) for the shape optimal design of concrete gravity dams including hydrodynamic effects. In the work of Slajegheh and Khosravi [12], the shape optimal design of concrete gravity dams including the dam–reservoir–foundation rock interaction was obtained using the hybrid of GSA and PSO. Khatibinia and Khosravi [13] introduced a hybrid approach based on an improved gravitational search algorithm (IGSA) and orthogonal crossover (OC) for the optimal shape design of concrete gravity dams. Deepika and Suribabu [14] used the differential evolution technique for the optimal design of gravity dam. Kaveh and Zakian [15] presented the shape optimization of a gravity dam imposing stability and stress constraints. In new recently, Khatibinia et al. [16] introduced the optimal shape design of concrete gravity dams considering dam–reservoir–foundation interaction and nonlinear effects subject to earthquake. In this study, the geometry and material nonlinearity effects of dam were considered in the analysis procedure of dams.

The focus of this paper is on the RBDO of concrete gravity dams subjected to earthquake load using subset simulation. In order to achieve this purpose, firstly, a Finite Element (FE) model of a concrete gravity dams including the material nonlinearity effects and the dam–reservoir–foundation rock interaction is constructed for obtaining dynamic responses in time domain and generating several accurate sampling points. Secondly, a version of the support vector machine (SVM) approach as an efficient metamodel is provided by utilizing these sampling points to replace the time consuming dynamic analysis of FE model. At last, this approximate model is utilized during the optimum process in order to obtain the final best shape of dam considering uncertainties. In the optimization process, the optimal shape of concrete gravity dam is found by minimizing the total cost of concrete gravity dam for the failure probability of dam as constraint. In this study, subset simulation with Markov Chain Monte Carlo (MCMC) sampling as an advance tool in the reliability approach is utilized to

accurately estimate the failure probability of dam with a minimum number of samples. The RBDO problem is solved using the hybrid of IGSA and OC proposed by Khatibinia and Khosravi [10]. The results demonstrate that the RBDO approach in comparison with the deterministic optimum approach maintain a good balance between the safety and cost in the shape optimal design of concrete gravity dams.

2. DEFINITION OF THE RBDO PROBLEM

The reliability–based design optimization (RBDO) is considered as a common practice for many structural engineering systems. The main aim of the RBDO approach is to minimize the total cost whereas ensure a given target reliability is achieved with respect to uncertainties in structural parameters and operating conditions. In the RBDO problem, variables consist of two vectors: design vector and random vector. The components of the design vector $\mathbf{X}=[x_1, x_2,\dots x_n]$ represent the design variables. The random vector $\mathbf{Z}=[z_1, z_2,\dots z_q]$ is utilized for the description of uncertain variables (or intervening random parameters).

The RBDO problems are formulated in several different classifications. Hence, the following formulation is a special case that expresses the purposes of this contribution:

$$\begin{aligned}
&\text{Minimize}: \ C_T(\mathbf{X},\mathbf{Z}) \\
&\text{Subject to}: P_f(\mathbf{X},\mathbf{Z}) \leq P_{f,\text{Target}} \\
&\qquad\qquad \mathbf{X} = \{x_1, x_2, \dots, x_i, \dots, x_n\}
\end{aligned} \tag{1}$$

where $C_T(\mathbf{X},\mathbf{Z})$ is the total cost; $P_f(\mathbf{X},\mathbf{Z})$ is the system failure probability; $P_{f,\text{Target}}$ is the target failure probability; x_i represents the ith vector element in $\mathbf{X}$.

In this study, the constraint are handled by using the the external penalty function as one of the most common forms of the penalty function as follows [17–20]:

$$\tilde{f}(\mathbf{X},\mathbf{Z}) = \begin{cases} C_T(\mathbf{X},\mathbf{Z}) & \text{if } \mathbf{X} \in \tilde{\Delta} \\ C_T(\mathbf{X},\mathbf{Z})\left[1 + r_p\, f_p(\mathbf{X},\mathbf{Z})\right] & \text{otherwise} \end{cases} \tag{2}$$

where $\tilde{f}$, f_p and r_p are the modified function, the penalty function, and an adjusting coefficient, respectively. Also, $\tilde{\Delta}$ denotes the feasible search space. The penalty function based on the violation of normalized constraints is defined as:

$$f_p(\mathbf{X},\mathbf{Z}) = \max\left[\frac{P_f}{P_{f,\text{Target}}} - 1, 0.0\right]^2 \tag{3}$$

2.1 Total cost

The total cost, C_T, consists of the initial structural cost (i.e. material costs) and the expected failure cost, which is defined by the product of failure cost and failure probability. The

failure cost may encompass the costs of repairing or replacing the damaged structure, removing the collapsed structure, and compensation for injury or death of general users. Thus, the total cost can be expressed as a function of the design vector **X** and the random vector **Z** as follows:

$$C_T(\mathbf{X}, \mathbf{Z}) = C_I(\mathbf{X}) + P_f(\mathbf{X}, \mathbf{Z}) C_f \tag{4}$$

where C_I and C_f are the initial cost of structure and the failure cost, respectively. The initial cost of a concrete gravity dam is obtained as:

$$C_I(\mathbf{X}) = C_c \rho_c g V \tag{5}$$

where C_c, ρ_c and V are the unit cost coefficient of concrete, the mass density of concrete and the volume of gravity dam, respectively. g is gravity acceleration.

2.2 The failure probability of concrete gravity dam

The failure probability, $P_f(\mathbf{X}, \mathbf{Z})$, as the probabilistic constraint can be described by the performance function [21]:

$$P_f(\mathbf{X}, \mathbf{Z}) = \Pr[g(\mathbf{X}, \mathbf{Z}) \le 0] \tag{6}$$

where $\Pr[.]$ is the probability of system failure and $g(\mathbf{X}, \mathbf{Z})$ is the performance function, which indicates failure for a given design **X** and realization of **Z** when it is less than or equal to zero. In other words, the performance function (limit state function) separates the variable space into a safe region for which $g(\mathbf{X}, \mathbf{Z}) > 0$ and a failure region where $g(\mathbf{X}, \mathbf{Z}) \le 0$.

In the present study, the behavior and stability constraints are considered as the performance function. The behavior performance function consist on the principal stresses in the body of gravity dam which is defined as follows [22]:

$$g_1 = \sigma_{\max}^T - \bar{\sigma}_T \tag{7}$$

$$g_2 = \left| \sigma_{\max}^C \right| - \bar{\sigma}_c \tag{8}$$

where $\sigma_{\max}^T$ and $\sigma_{\max}^C$ are the maximum tensile and compressive principal stresses of dam body due to dam–reservoir–foundation system subjected to earthquake load, respectively. $\bar{\sigma}_T$ and $\bar{\sigma}_C$ are the allowable tensional and compressive stresses.

The stability performance function of a gravity dam are defined in terms of its factors of safety against sliding, overturning and uplift pressure, respectively. The factor of safety against sliding is equal to the ratio of the total frictional force, F_V, which the foundation can develop to the force tending, F_H, to cause sliding as follows [22]:

$$g_3 = 1.75 - \frac{\sum F_V}{\sum F_H} \tag{9}$$

The factor of safety against overturning about the toe is defined as the ratio of the resisting moments, M_R, to the overturning moments, M_O, which is considered as stability constraint. It is expressed as [22]:

$$g_4 = 1 - \frac{\sum M_R}{\sum M_O} \tag{10}$$

Furthermore, the safety of concrete gravity dam against uplift pressure force is considered as follows [22]:

$$g_5 = 1 - \frac{B}{6e} \tag{11}$$

where B and e are the bottom width of the dam eccentricity of the resultant force on the dam section, respectively.

Therefore, the failure probability represented in Eq. (6) is inversely related to the reliability of a concrete gravity dam, which is an integrated result of individual component reliabilities, assuming series. In a series system, the failure is happened when any of the performance functions fails. Thus, the failure probability of a dam is defined as [23]:

$$P_f(\mathbf{X}, \mathbf{Z}) = \Pr\left[\bigcup_i g_i(\mathbf{X}, \mathbf{Z}) \le 0\right] \tag{12}$$

3. RELIABILITY ANALYSIS

3.1 Subset simulation

Based on the failure event $F = \{\mathbf{Z} : g(\mathbf{Z}) < 0\}$, the probability density function (PDF) of $\mathbf{Z}$ is defined by $f_z(\mathbf{Z})$. Assume $b_1 > b_2 > \cdots > b_m = 0$ as a decreasing sequence of the threshold values of failure events $F_k = \{\mathbf{Z} : g(\mathbf{Z}) < b_k\}$, $(k = 1, 2, ..., m)$ shown in Fig. 1. Then the failure events satisfy the following relations [24–27]:

$$F_1 \supset F_2 \supset \cdots \supset F_m = F \tag{13}$$

and
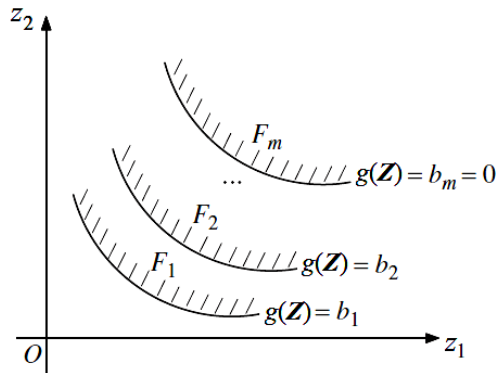
$$F_k = \bigcap_{i=1}^{k} F_i \tag{14}$$

Figure 1. A set of failure events in subset simulation

According to the multiplication theorem and the definition of conditional probability in the probability theory, the following equation holds:

$$P_f = P(F) = P(F_m \mid F_{m-1})P(F_{m-1}) = \cdots = P(F_1)\prod_{i=2}^{m}P(F_i \mid F_{i-1}) \tag{15}$$

Eq. (15) expresses the failure probability as a product of a sequence of conditional probabilities $P(F_i \mid F_{i-1}), (i = 2, 3, ..., m)$ and $P(F_1)$. The idea of subet simulation is to obtain the failure probability P_f by estimating these conditional probability quantities. By defining $P_1 = P(F_1), P(F_i \mid F_{i-1}), (i = 2, 3, ..., m)$. The failure probability in Eq. (15) can be expressed by:

$$P_f = \prod_{i=1}^{m}P_i \tag{16}$$

Generally, the value of P_f is small in practical engineering and cannot be estimated efficiently by a numerical simulation. By choosing the intermediate failure events F_i $(i = 1, 2, ..., m-1)$ appropriately, conditional probabilities involved in Eq. (16) can be made sufficiently large so that they can be evaluated efficiently by simulation procedures.

3.2 Subset simulation with MCMC

MCMC algorithm has been introduced for computing conditional failure probabilities by using Markov chain samples with limiting stationary distribution $q(\mathbf{Z} \mid F_{i-1})$ (i=2,3, ..., m) . For this purpose, the Metropolis–Hastings Criterion is employed to draw the Markov chain samples. The subet simulation with MCMC proceeds as in the following [28]:

1. Generate N_1 samples i.e. $\mathbf{Z}_k^{(1)}(k = 1, 2, ..., N_1)$ of the probability density function (PDE) $f_Z(\mathbf{Z})$ by direct MCS for i=1.

2. Compute the corresponding response value $g(\mathbf{Z}_k^{(1)})(k = 1, 2, ..., N_1)$. The first intermediate threshold value b_1 is adaptively chosen as the $(p_0 N_1)$th (p_0 is a pre–established conditional

probability level, such that $p_0 = 0.1$, and $p_0 N_1$ should be chosen as an integer value in the descending list of N_1 response values. Then the first intermediate failure event is defined by $F_1 = \{\mathbf{Z} : g(\mathbf{Z}) < b_1\}$. So failure probability $P_1 = P(F_1)$ can be estimated by $\hat{P}_1 = p_0$.

3. Start from these $p_0 N_{i-1}$ conditional samples that lie in F_{i-1} for ith level $(i = 2, ..., m)$. MCMC simulation is used to generate $(N_i - p_0 N_{i-1})$ additional conditional samples following the conditional PDF $q(\mathbf{Z}|F_{i-1})$. Then, a total N_i conditional samples $\mathbf{Z}_k^{(i)} (k = 1, 2, ..., N_i)$ are made up for the ith level $(i = 2, ..., m)$.

4. Compute the corresponding response values $g(\mathbf{Z}_k^i)(k = 1, 2, ..., N_i)$. The intermediate threshold value b_i is adaptively chosen as the $(p_0 N_i)$th value in the descending list of N_i response values. Then the next intermediate failure event is defined by $F_i = \{\mathbf{Z} : g(\mathbf{Z}) < b_i\}$. The conditional failure probability $P_i = P(F_i | F_{i-1})$ can be estimated by $\hat{P}_i = p_0$, and the estimator of the failure probability $P(F_{i-1})$ is given by $\hat{P}(F_{i-1}) = \prod_{j=1}^{i-1} \hat{P}_j$.

5. Repeat the steps (3) and (4) until the mth adaptively chosen threshold value b_m is less than zero. Then, assume $b_m = 0$, and the target failure probability level $P(F) = P(F_m)$ is reached. The conditional failure probability $P_m = P(F_m | F_{m-1})$ can be estimated by $\hat{P}_m = N_f / N_m$, where N_f is the number of samples that lie in the target failure event $F = F_m$. The target failure probability $P_f = P(F) = P(F_m)$ can be estimated by:

$$\hat{P}_f = p_0^{(m-1)} \times \frac{N_f}{N_m} \tag{17}$$

4. GEOMETRICAL MODEL OF CONCRETE GRAVITY DAMS

In order to optimize the shape of a concrete gravity dam depicted in Fig. 2, the geometrical model of a dam can be assigned by the seven parameters as follows:

$$\mathbf{X} = \{\bar{B} \ B_1 \ B_2 \ B_3 \ H_2 \ H_4 \ H_5\} \tag{18}$$

where $\bar{B}$ and H_1 are two parameters required to defined crest and free board of gravity dam, respectively. H_3 depends on H_4 and reservoir water level (H).

5. FORMULATION OF DAM–RESERVOIR–FOUNDATION SYSTEM

The formulation of the coupled dam–reservoir–foundation based on the finite element method (FEM) approach is impilemented according to Refs [13, 29, 30]. In this approach, displacements are selected as the variables in both fluid and structure (i.e. dam and foundation) domains. Fluid is assumed to be linearly elastic, inviscid and irrotational. In this

study, the dam–water–foundation system is simulated as a 2–D model. The nonlinear behavior of dam concrete is idealized as an elasto–plastic material via the associative Drucker–Prager model [31, 32]. This model with angle of internal friction (ϕ) and cohesion factor (C) has been proposed for frictional materials such as concrete and utilized for an approximation to the Mohr-Coulomb law. In the analysis phase of concrete gravity dam, first, a static analysis of concrete gravity dam–reservoir–foundation system is initially implemented under a gravity load and a hydrostatic pressure, and then the nonlinear dynamic analysis of dam–reservoir–foundation system is performed using Newmark–Beta integration method [33]. After that, the principal stresses at elements of the dam body are evaluated using nodal relative displacement of the gravity dam.
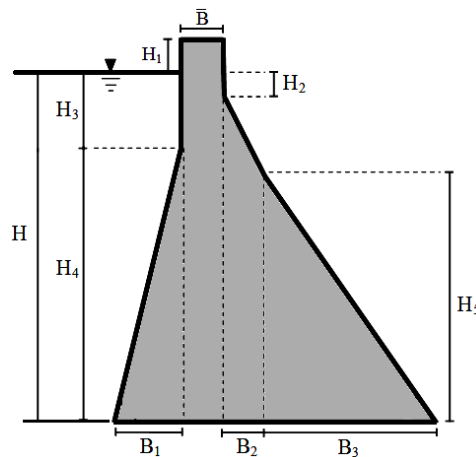

Figure 2. Geometrical model of concrete gravity dam

6. THE METAMODEL TECHNIQUE FOR RELIABILITY ANALYSIS

In the RBDO problem of structures the computational effort becomes excessive due to the enormous sample size required for the reliability analysis. This drawback can increase when the dynamic responses of structure obtained using FEM are required in the reliability analysis. In order to reduce the computational effort, metamodel techniques have been developed to predict structural responses in reliability process [2, 3]. Hence, approximating the structural responses can effectively reduce the computational burden. In this study, instead of the FE dynamic analysis of dam–reservoir–foundation system the weighted least squares support vector machine (WLS–SVM) as a metamodel technique is utilized to predict the maximum principle stresses of concrete gravity dams in the RBDO process. The successfull applications of the WLS–SVM approach have been reported in Refs. [2, 3, 11–13, 34].

6.1 Theory of the WLS–SVM

The WLS–SVM approach introduced by Suykens *et al*. [35] has been proposed for modeling the high non–linear system based on small sample. The WLS–SVM is presented as the optimization problem in primal weight space as follows [35]:

$$\text{Minimize } J(\boldsymbol{\omega},\xi) = \frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{\omega} + \frac{1}{2}\gamma\sum_{i=1}^{n} w_i \xi_i^2$$

$$\text{Subjest to } y_i = \boldsymbol{\omega}^T\varphi(\boldsymbol{x}_i) + b + \xi_i, \qquad i = 1,2,...,n \tag{19}$$

with $\{\boldsymbol{x}_i, y_i\}_{i=1}^{n}$ a training data set, input data $\boldsymbol{x}_i \in R^n$ and output data $y_i \in R$. $\varphi(.):R^n \to R^d$ is a function which maps the input space into a higher dimensional space. The vector $\boldsymbol{\omega} \in R^d$ represents weight vector in primal weight space.

The symbols $\xi_i \in R$ and $b \in R$ represent error variable and bias term, respectively. The Lagrange multiplier method is utilized for solution of the dual problem (i.e. Eq. (19)) as:

$$L(\boldsymbol{\omega},b,\xi,\boldsymbol{\alpha}) = J(\boldsymbol{\omega},\xi) - \sum_{i=1}^{n}\alpha_i(\boldsymbol{\omega}^T\varphi(\boldsymbol{x}_i) + b + \xi_i - y_i) \tag{20}$$

Based on the Karush–Khun–Tucker (KKT) conditions, after optimizing Eq. (20) and eliminating $\boldsymbol{\omega}$ and ξ, the solution is given by the following set of linear equation:

$$\begin{bmatrix} \Omega + V_\gamma & I_n^T \\ I_n & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{\alpha} \\ b \end{bmatrix} = \begin{bmatrix} \boldsymbol{y} \\ 0 \end{bmatrix} \tag{21}$$

where $V_\gamma = diag\{1/\gamma\bar{v}_1,...,1/\gamma\bar{v}_n\}$; Ω is $n{\times}n$ Hessian vector, which expression is: $\Omega_{i,j} = \langle\varphi(\boldsymbol{x}_i),\varphi(\boldsymbol{x}_j)\rangle_H = K(\boldsymbol{x}_i,\boldsymbol{x}_j)$. $K(.,.)$ is a kernel, which in this study, radial basis function (RBF) is selected as the kernel function of WLS–SVM as follows:

$$K(\boldsymbol{x}_i,\boldsymbol{x}_j) = \exp\left(-\frac{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2}{2\sigma^2}\right) \tag{22}$$

Therefore, the resulting WLS–SVM model for the prediction of functions becomes:

$$y(\boldsymbol{x}) = \sum_{i=1}^{n}\alpha_i K(\boldsymbol{x}_i,\boldsymbol{x}) + b \tag{23}$$

Here, $\boldsymbol{\alpha}$ and b is the solution to the problem (21). The parameters represent the high dimensional feature spaces that is non–linearly mapped from the input space $\boldsymbol{x}$. Furthermore, predicting value of $\boldsymbol{x}$ is obtained by the model peresented in Eq. (23). The structure of the WLS–SVM metamodel is shown in Fig. 3.
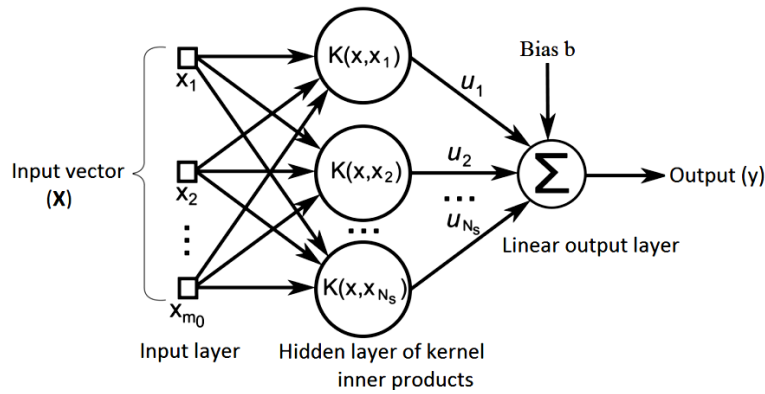
Figure 3. The structure of the WLS–SVM metamodel

6.2 Designing of the WLS–SVM for predicting the dynamic responses of dam

In order to employ the WLS–SVM metamodel during the RBDO process, the WLS–SVM metamodel is trained by using a randomly generated database which consists of the combinations of the design variables, the random variables and the seismic responses of concrete gravity dams. In this case, the design variables and the random variables as inputs of the WLS–SVM are considered as follows:

$$I_{WLS-SVM} = \left\{ (X,Z)_1, (X,Z)_2, ..., (X,Z)_n \right\}$$ (24)

and the maximum tensile and compressive principle stresses of concrete gravity dams as outputs of the WLS–SVM are selected as:

$$O_{WLS-SVM} = \left\{ (\sigma_{max}^T, \sigma_{max}^C)_1, (\sigma_{max}^T, \sigma_{max}^C)_2, ..., (\sigma_{max}^T, \sigma_{max}^C)_n \right\}$$ (25)

In this study, the WLS–SVM with the 10–fold cross–validation (CV) is employed to find the optimal values of γ and σ for training the WLS–SVM model. For training and testing of the WLS–SVM based on the RBF kernel function, the following process is implemented:

(1) A database for training and testing the WLS–SVM model defined Eq. (24) is randomly generated.

(2) For each concrete gravity dam corresponding to a input vector in database FEA is performed, and the maximum tensile and compressive principle stresses of dams as the seismic responses of dam are obtained. The seismic responses is considered as the output of the WLS–SVM.

(3) The provided database is divided to training and testing sets on a random basis.

(4) Two WLS–SVM models are trained and tested based on the generated sets for predicting the maximum tensile and compressive principle stresses of dams.

To validate the performance of the WLS–SVM model, the mean absolute percentage error (*MAPE*), the relative root–mean–squared error (*RRMSE*) and the absolute fraction of variance (R^2) arose during tesing process of the WLS–SVM are calculated as:

$$MAPE = \frac{1}{n_t} \sum_{i=1}^{n_t} 100 \times \left| \frac{a_i - p_i}{a_i} \right|$$ (26)

$$RRMSE = \sqrt{\frac{n_t \sum_{i=1}^{n_t}(a_i - p_i)^2}{(n_t - 1)\sum_{i=1}^{n_t} a_i^2}} \tag{27}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n_t}(a_i - p_i)^2}{\sum_{i=1}^{n_t} p_i^2} \tag{28}$$

where a and p are the actual value and the predicted value, respectively; and n_t is the number of testing samples. It is noted that the smaller *RRMSE* and *MAPE* and the larger R^2 are indicative of better performance generality.

7. NUMERICAL RESULTS

In the present study, the RBDO of Pine Flat dam located on King's River near Fresno California as a real–world structure is investigated in order to obtain its optimal shape subjected to earthquake loads using subset simulation. The properties of the dam structure are 400 *ft* height with a crest length of 1840 *ft* and its construction about 9491.94 *kip* concrete [36]. The lower and upper bounds of the design variables (**X**) defined in Section (4) are considered to find the optimal shape of the Pine flat dam. The bounds of the variables are shown in Table 1 [36]:

Table 1: The lower and upper bounds of design variables

Design variable	Lower bound (*ft*)	Upper bound (*ft*)
$\bar{B}$	16.67	39.34
B_1	30.232	34.166
B_3	28.413	34.727
B_4	210.6	257.4
H_2	12.6	15.4
H_4	302.32	341.66
H_5	270	330

The optimal shape of this dam is found subjected to the S69E component of Taft Lincoln School Tunnel during Kern country, California, earthquake (July 21, 1952) [36]. This component of the recorded ground motion is shown in Fig. 4.

To demonstrate the effect of the RBDO approach in the optimal shape of the selected dam, two cases of optimization by considering the nonlinear effects of dam–reservoir–flexible foundation rock interaction are considered and compared as follows:

Case 1: The optimal shape of the dam based on the RBDO approach.

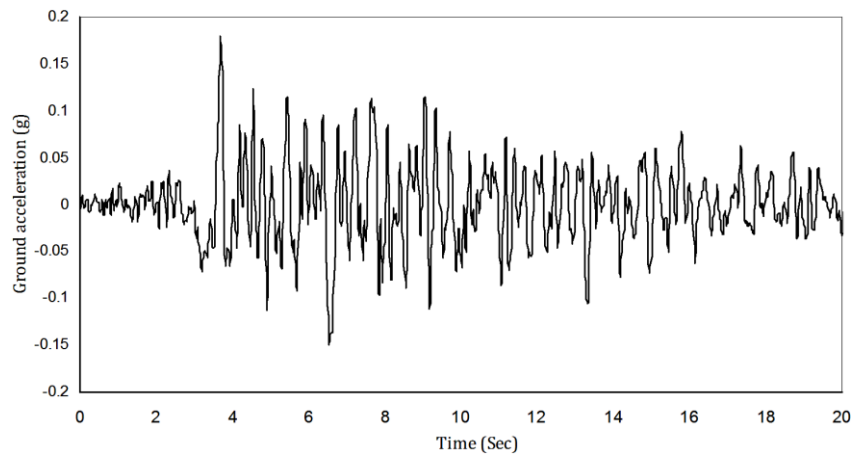Case 2: The optimal shape of the dam based on the deterministic optimum approach.

Figure 4. Ground motion at Taft Lincoln Tunnel; Kern country, California, 1952

7.1 Validation of coupled dam–reservoir–foundation rock system

In order to obtain the dynamic responses of the dam in time domain and the RBDO process of the dam, a FE model of the dam including the material nonlinearity effects and the dam–reservoir-foundation rock interaction is required. For achieving this purpose, an idealized model of Pine Flat gravity dam–reservoir–foundation rock system for full reservoir is constructed using FEM and depicted on Fig. 5.
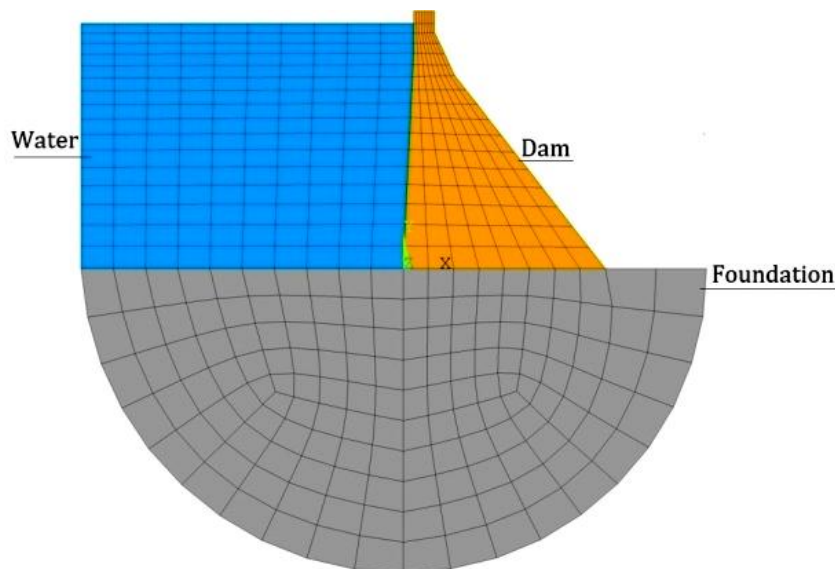
Figure 5. The FE model of Pine Flat dam with reservoir and foundation rock

The properties of concrete of dam body, water and foundation are given in Table 2 [36]. In order to validate the FE model of the dam–reservoir–foundation rock system with the employed assumptions in this study, the first natural frequency of the FE model of Pine Flat dam for four cases are obtained. The results of the FE model are compared with those reported by Chopra and Chakrabarti [36] and given in Table 3.

Table 2: The material properties of the dam, water and foundation rock

Material	Property	Value
Concrete		
	Modulus of elasticity (*psi*)	3.25×10^6
	Poisson's ratio	0.2
	Mass density (*lb/ft³*)	155
Water		
	Mass density (*lb/ft³*)	62.4
	Wave velocity (*ft/sec*)	4720
	Wave reflection coefficient	0.817
Foundation		
	Moulus of elasticity (*psi*)	10^7
	Poisson's ratio	0.33

As observed from the results of Table 3, a good conformity has been achieved between the results of the present study with those reported in the literature.

Table 3: A comparison of the first natural frequencies from the literature with FEM

Case	Foundation rock condition	Water	Natural frequency (Hz)		Error (%)
			Chopra and Chakrabarti [36]	The present study	
1	Rigid	Empty	3.1546	3.152	0.082
2	Rigid	Full	2.5189	2.525	0.242
3	Flexible	Empty	2.9325	2.93	0.085
4	Flexible	Full	2.3310	2.383	2.18

It can also be observed from Table 3 that when the reservoir is empty and the foundation is rigid (Case 1) the first frequency of the dam is maximal. Furthermore, a minimum value for the first frequency is obtained when the dam–water–foundation rock interaction (Case 4) is considered.

7.2 Intervening random parameters

In this study, the properties of concrete, the water level of the reservoir and the friction cofficient between the dam and its foundation are considered as the intervening random parameters for the RBDO of the selected dam. The probability density function (PDE), mean value and standard deviation for the water level of the reservoir (*H*), the allowable tensional stress $(\bar{\sigma}_T)$, the allowable compressive stress $(\bar{\sigma}_C)$ and the friction cofficient (μ) are listed in Table 4. The PDE, the maximum and minimum values for the angle of internal friction (ϕ), the cohesion factor (*C*) and the Modulus of elasticity (*E*) for concrete of the dam are also given in Table 5. It is noted that theses uncertain properties of the parameters are selected from the studies reported in the literature [38, 39].

Table 4: The PDE, mean and standard deviation of the random parameters

Variable	PDE	Mean value	Standard deviation
$H\,(ft)$	Normal	381	1.64
$\bar{\sigma}_T\,(Klb/ft^2)$	Normal	31.33	1.25
$\bar{\sigma}_C\,(Klb/ft^2)$	Normal	261.06	20.88
μ	Normal	0.75	0.03

Table 5: The PDE, the minimum and maximum values mean of the random parameters

Variable	PDE	Minimum value	Maximum value
$C(Klb/ft^2)$	Uniform	14.62	18.8
ϕ°	Uniform	32	45
$E(Klb/ft^2)$	Uniform	438594	480365

7.3 Generation of a database

In order to train and test the WLS-SVM metamodel, a discrete database should be generated randomly. This database is considered as input of the WLS–SVM. The output of the WLS–SVM consists of the maximum tensile and compressive principal stresses of dam body due to dam–reservoir–foundation system subjected to earthquake load. To achieve this purpose, first, 200 combinations of the design variables (**X**) shown in Table 1 and the intervening random variables (**Z**) reported in Tables 3 and 4 are generated using Latin Hypercube Design (LHD) sampling proposed for computer experiments [40]. Then, the nonlinear dynamic analysis of dam–reservoir–foundation system using FEA is performed for each of the combinations, and the maximum tensile and compressive principal stresses of dam body are obtained.

7.4 Training and testing the metamodel

To predict the maximum tensile and compressive principal stresses of dam body during the RBDO process of the selected dam, two metamodels are trained based on the generated database. To achieve this purpose, the samples are selected on a random basis and from which 70% and 30% samples are employed to train and test the WLS–SVM metamodel. The results of testing the performance generality of the WLS–SVM models based on the statistical values of *MAPE*, *RRMSE* and R^2 are reported in Table 6:

Table 6: The statistical values for the WLS–SVM model in testing mode.

The WLS–SVM metamodel	Statistical values		
	MAPE	*RRMSE*	R^2
For the maximum tensile principal stress	4.71	0.0560	0.9984
For the maximum compressive principal stress	10.07	0.3273	0.9397

All of the statistical values in Table 6 demonstrate that the WLS–SVM metamodels achieve a good performance generality in predicting the time history responses of the gravity

dam samples. Thus, the WLS–SVM metamodels instead of the original time consuming dynamic analysis of FE model not only are efficiently replaced during the RBDO proces of the gravity dam but also significantly can reduce the computation cost of the RBDO process. Figs. 6 and 7 also show the agreement between the actual responses and those predicted with the WLS–SVM metamodel.
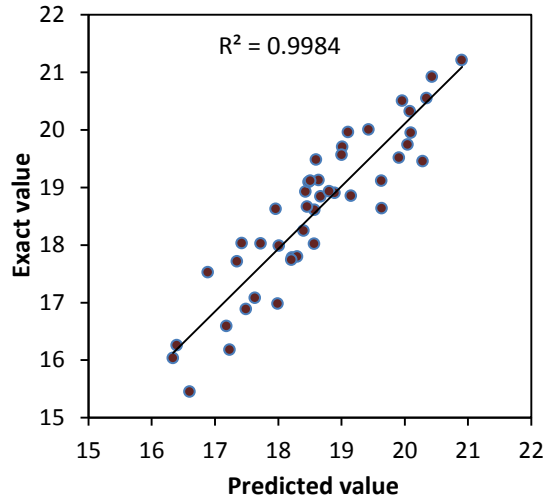

Figure 6. Absolute percentage errors associated with the maximum tensile principal stress
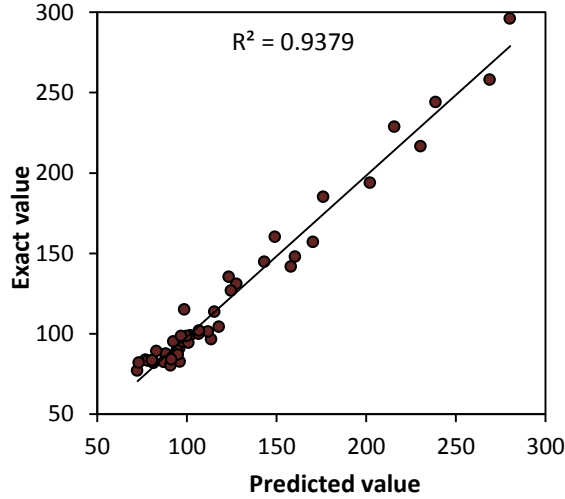
Figure 7. Absolute percentage errors associated with the maximum compressive principal stress

The displayed results in these figures demonstrate that the WWLS–SVM models achieve a good performance generality in predicting the dynamic responses of the dam samples.

7.5 The results of the RBDO approach

In this study, the IGSA–OC algorithm proposed by Khatibinia and Khosravi [13] is utilized for the RBDO process. The parameters of the IGSA–OC algorithm are also selected based

on the values proposed in the work of Khatibinia and Khosravi [13]. The parameters of the subset simulations are chosen as $N_1 = 1000$ and $p_0 = 0.1$. The unit cost coefficient of concrete (C_c) is assumed to be 60$/*Klb* and the failure cost (C_f) is estimated to be 5×10^6\$.

In order to consider the stochastic nature of the optimization process, ten independent optimization runs are performed for the selected dam and the four best solutions are reported in Table 7.

Table 7: Optimal solutions of concrete gravity dam based on the RBDO approach

Variables	Optimization design			
	Case 1	Case 2	Case 3	Case 4
$\bar{B}$ (*ft*)	36.44	39.34	36.65	34.97
B_1 (*ft*)	33.70	34.16	33.73	32.16
B_2 (*ft*)	32.22	34.37	33.76	33.37
B_3 (*ft*)	256.20	257.40	248.88	255.83
H_2 (*ft*)	14.58	14.71	15.08	13.31
H_4 (*ft*)	327.28	332.15	330.02	326.13
H_5 (*ft*)	323.55	312.43	326.59	322.41
Concrete weight (*Klb*)	11261	11359	11246	11158
Mean of the best solutions	11256			

To demonstrate the promising results of the RBDO procedure, the optimal shape of the dam is implemented based on the deterministic optimum approach involving the material nonlinearity effects and dam–reservoir–foundation interaction. The results of the deterministic optimum approach is given in Table 9. It is noted that the results has been reported in the work of Khatibinia *et al*. [16].

Table 8: Optimal solutions of concrete gravity dam for the deterministic optimum approach

Variables	Optimization design			
	Case 1	Case 2	Case 3	Case 4
$\bar{B}$ (*ft*)	34.67	33.49	38.76	33.69
B_1 (*ft*)	34.16	34.16	34.17	34.17
B_2 (*ft*)	33.60	34.73	32.96	32.6418
B_3 (*ft*)	257.40	250.86	248.31	257.40
H_2 (*ft*)	15.40	15.40	15.40	15.18
H_4 (*ft*)	336.33	341.66	338.00	341.66
H_5 (*ft*)	312.90	320.45	312.64	311.79
Concrete weight (*Klb*)	11049	11058	11047	10928
Mean of the best solutions	11020.5			

For the best result of the RBDO approach and the deterministic optimum (DO) approach given in Tables 7 and 8, the failure probability (P_f), the initial cost (C_I), the failure cost (C_R) and the total cost (C_T) of the selected dam are listed in Table 9.

Table 9: Optimal solutions of concrete gravity dam for the nonlinear effects

Case	P_f	C_I ($)	C_R ($)	C_T ($)
RBDO approach	0.00017	669480	850	670330
DO approach	0.00650	655680	32500	688180

By comparing the optimum solution based on the RBDO approach with that of the DO approach, not only the total cost of the optimum solution based on the RBDO approach is less than that of the DO approach, but also the safety of the dam is considerably increased. In other words, the RBDO process of the dam can efficiently obtains a balance between the total cost and safety of the dam.

9. CONCLUSIONS

This contribution has introduced the RBO approach of concrete gravity dams subjected to earthquake load using subset simulation. The RBDO problem is formulated such that the optimal shape of concrete gravity dam described by a number of variables is obtained by minimizing the total cost of concrete gravity dam for the failure probability of dam as constraint. In order to achieve this purpose, firstly, a model of dam–reservoir–foundation rock interaction with the material nonlinearity effects is constructed based on FEM. Secondly, the WWLS–SVM approach as an efficient metamodel is trained and tested to replace the time consuming dynamic analysis of the FE model in the RBDO process. Subset simulation with MCMC sampling is utilized to estimate accurately the failure probability of dams with a minimum number of samples.

The optimal results show that the RBDO approach is a more rational method and maintain a good balance between the safety and cost of dam. Thus, the probabilistic approach is more appropriate than a deterministic approach for the design and optimization of concrete gravity dams that rely on dam–reservoir–foundation rock interaction. It is noted that further research efforts will focus on the characteristics associated with ground motions in the RBDO approach of concrete gravity dams.

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