AN EFFECTIVE METHOD FOR SIMULTANEOUSLY CONSIDERING TIME-COST TRADE-OFF AND CONSTRAINT RESOURCE SCHEDULING USING NONLINEAR INTEGER FRAMEWORK

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ABSTRACT

Critical Path Method (CPM) is one of the most popular techniques used by construction practitioners for construction project scheduling since the 1950s. Despite its popularity, CPM has a major shortcoming, as it is schedule based on two impractical acceptance that the project deadline is not bounded and that resources are unlimited. The analytical competency and computing capability of CPM thus need to be enhanced by applying some additional techniques like Time-Cost Trade-off (TCT) and Constraint Resource Scheduling (CRS) separately after the initial schedule is determined. Therefore, this paper is focusing on an effective method for considering simultaneously TCT and CRS using a nonlinear integer framework, taking help of Microsoft Project Software (MSP) and Microsoft Excel Solver. Through this method, first, a start delay technique is applied to the baseline schedule to level out the resource over allocation and then the project network diagram is modified according to the resource-leveled schedule. Secondly, a time-cost optimization is used over the resource-leveled schedule network diagram, using MS Excel solver to get the optimum duration associated with the minimum total cost of the project satisfying resource constraint. The proposed framework using overtime for activity expedition, and required less time to generate the final solution compare to the available methods considering TCT+CRS simultaneously.

Keywords: time cost trade-off; constrained resource scheduling; overtime working; labor law; nonlinear integer programming; project total cost.

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1. INTRODUCTION

There are several scheduling techniques, such as Gantt charts, Critical Path Method (CPM), and Program Evaluation and Review Technique (PERT) used in construction projects. Among these techniques, CPM has been widely used by construction practitioners since 1950s. Despite its popularity, CPM does not accommodate deadline and resource constraints. Therefore, additional techniques (TCT & CRS) have to be implied to overcome CPM's incapability regarding unrestricted deadline and unlimited resources assumptions. The goal of the Constrained Resource Scheduling (CRS) is to find an optimal or near-optimal solution (base on the applied method, exact or heuristic) to resolve the resource over-allocation, even at the expense of increased project duration. The objective of Time-Cost Trade-off (TCT) is to specify the optimum duration correlated with the minimum total cost (direct and indirect costs). Generally, the project's total cost decreases by crashing critical activities until it grasps a certain project duration where the total cost starts to increase, this is going to be accepted as the optimum duration point, which is associated with minimum project total cost. For project managers, this is the valuable duration for the project as it allows project completion at the minimum possible total cost. Furthermore, early accomplishment allows project managers to divert resources to other projects, undertake new projects, advance reputation, and strengthen relationships with the client.

   The available literature describe several techniques to address the Time-Cost Trade-off (TCT) and Constrained Resource Scheduling (CRS) Independently, little effort has been devoted to considering them simultaneously because of the higher modeling complexity involved. Therefore, this paper is focusing effective method for considering TCT+CRS simultaneously and finding the minimum possible project total cost satisfying resource constraint. The framework is using overtime for activity expedition (crashing) and considering labor law requirements as well.

2. TIME-COST TRADE-OFF

One of the assumptions in CPM is that the duration of an activity can be reduced or crashed to a certain extent by increasing the resources assigned to it. As is known, the execution of an activity involves both direct costs and indirect costs. Any reduction in duration (by increasing resources, using advanced materials or increasing working hours) of critical path's activities can reduce the project duration and, thereby, enhance the possibility of a reduction in project cost. So some activities along the critical path sometimes need to be shortened in order to reduce the overall duration of the project. This leads to a decrease in the indirect expenses (due to a decrease in duration) and increase in the direct expenses (due to more mobilization of resources, using advanced materials or increasing working hours). So, time/Cost Tradeoff (TCT) is defined as a process to identify suitable construction activities for speeding up, and for deciding 'by how much' to get the best possible savings in both time and cost. It is a technique used to overcome critical path method's (CPM) lack of ability to confine the schedule to a specified duration. Isidore & Back (2002), proposed an upgraded model in which flexibility of activities in terms of time and cost is taken into account. [1]. Rana A. Al Haj (2015), developed an NLIP model that is going to solve time-

cost optimization taking into account the flexibility of the schedule. [2]. Mojtaba Maghrebi (2013), present a deterministic mathematical model for time-cost trade-off. This method relies on path constraints in a network while other similar methods are developed based on activities [3].

However, for projects involving a large number of activities with varying construction options, finding optimal TCT decisions becomes difficult and time-consuming. Finding optimum or near optimum solution for the time-cost trade-off problem involves the use of certain techniques such as manual time-cost tradeoff (TCT) techniques, mathematical techniques or Meta-heuristic techniques.

3. CONSTRAINT RESOURCE SCHEDULING (CRS)

Resource management is an important factor that gives the ability to construction contractors to remain competitive in today's construction business environment. Appropriate resource management assures to keep the project within budget and schedule. While preparing the network diagram we concentrated mainly on the technological constraints (for instance, one activity cannot start until the other is over), and assumed that the resources are unlimited. Further, the resources can be mobilized on demand anytime we require them. Although such ideal conditions may prevail in some exceptional projects, in most of the real-life projects we face resource constraints. In real-life situations, the activity start times not only have to deal with technological constraints in the form of their precedence relationships but also face the challenge of resource availability constraints. That is, we have limited quantity of resources at our disposal. The planning for scheduling of activities, thus, has to account for the different constraints that may be imposed on the availability of any of these resources, and ensure that sudden change in the requirement of these are avoided.

This problem (CRS) is one of the most complicated problems of operation research which has considerable progress in establishing exact solution and inventive methods at recent decades and recently many researchers addressed new optimization methods used to solve it.

Ming-Fung Francis (2015)**,** developed a modeling technique relying on the zero-one programming approach, considering to create optimum resource-constrained schedule satisfying time-dependent resource constraint. [4]. Stefan Creemers (2015), considered the uncertainty of activity duration to the basic RCPSP and called stochastic SRCPSP. [5]. Sanjay Tiwari (2015), introduces a two-step procedure where first, Microsoft excel software is used for TCT to find the various combination of activities durations (between normal and crash duration) which resulted with the targeted duration without considering resource constraints. All the schedules resulted in the first step are then exercised for applying CRS using Microsoft Project Software (MSP) to get the required project schedule meeting the deadline with available resources. [6]. Wail Menesi (2014), introduced a model which can be used for projects where the activities have more than one mode of execution with different resource requirements and called multimode resource-constrained project scheduling problem (MRCPSP). [7]. Ashraf Elazouni (2014), focused on the trade-off between cash, resources and profit [8]. Wenfa Hu (2013), used a genetic algorithm tool is used to evaluate the trade-off between time-cost-quality [9]. Hong Zhang (2012), focused on

minimizing project duration while considering multimode resource constraints [10]. Tarek Hegazi (2012), present construction scheduling optimization, satisfying both deadline and resource constraints. To solve these problems and generate fast, good solutions, this paper uses a heuristic method to meet both deadline and resource limits. The proposed method use crushing cycles of cheapest critical activities and resolve the resource over allocation during each cycle until the schedule meet both the resource constraints and deadline [11]. Seyed Hossein Hashemi Doulabi (2011), split the activities to level the resources in a very large project. Additional cast is considered for activity splitting and a trade-off between splitting cost and benefits which could be achieved for resource leveling is taken into account [12]. Min-Yuan Cheng (2015), present a new discrete optimization model for solving Multiple-Resources Leveling in Multiple Projects (MRLMP) [13]. Moncer Hariga (2011), presents an optimization model for resource leveling that allows activity splitting and minimizes its associated costs [14]. Gunnar Lucko (2011), focuses on resource leveling and investigates a novel resource model and its efficient optimization toward a leveled profile [15]. Siamak Baradaran (2011), presents a hybrid metaheuristic algorithm (HMA) for Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP) in PERT networks [16]. Dho Heon Jun (2011), presents the development of a novel multi-objective optimization model that is capable of measuring and minimizing undesirable resource fluctuations to maximize resource utilization efficiency and minimize project duration while complying with all precedence relationships and resource availability constraints [17].

Konstantinos Anagnostopoulos (2012), consider Resource-Constrained Critical Path Scheduling [18]. H. Taghaddos (2012), solve resource scheduling problems in large-scale construction projects. The overall objective is to maximizing the system's revenue or minimize total costs [19]. Li-hui Zhang (2013), proposes an optimization model based on the genetic algorithm in repetitive projects considering different requirements for resource continuity of activities [20].

4. PROPOSED NLIP FRAMEWORK FOR TCT+CRS USING OVERTIME

Assumptions: The following assumptions are accepted for stochastic analysis and compersion of the results in both cases of TCT+CRS using overtime and normal TCT+CRS.
- Activity's duration and cost are defined as a normal probability distribution with a mean and a standard deviation
- Duration of the activity equals its mean duration
- While crashing an activity per day, the activity duration will be decreased by one day, as well as the mean. The standard deviation is assumed to be equal in both cases
- 10000 iterations of Monte Carlo Simulation (MCS) are used for each run using @risk.

Model formulation

A nonlinear-integer programming formulation is used to find the minimum total project cost associated with the optimum duration in constraint resource conditions. The nonlinearity arises from the constraints over activities' relations and precedence. Activities' durations are assumed to be integer numbers to make it as real as possible.

Framework parameters

I = Project activities are denoted by the symbol i∈I where I is a set that comprises all project activities, and i = 1, 2, 3…, n.

P = Critical activities are denoted by the symbol p∈P, and p=1, 2, 3..., n.

Q = Non-Critical activities are denoted by the symbol q∈Q, and q=1, 2, 3…, n.

Faq = Total float of noncritical activity i before crashing.

Fbq = Total float of noncritical activity i after crashing.

NDi = Normal duration of activity i.

CDi = Crash duration of activity i.

CDRi = Current duration of activity i.

TC = Project total cost.

XT = Project targeted completion date.

IDC = Project indirect cost.

LTi,j = Lag time between activity i and the succeeding activity j.

COH = Project overhead cost per day.

PND = Penalty Days (Number of days beyond the deadline).

DCi = Direct cost of activity i (normal cost).

OWHi = Overtime Working Hours per labor for activity i.

CCi = Expenditure cost of activity i (crash cost).

NWHi = Normal Working Hours per labor for activity i.

CUCi = Activity i crashing unit cost / crashing slope.

NWi = Normal Wages per hour per labor for activity i.

NCi = Activity i normal cost; OF = Overtime Factor.

ORC = Overtime Cost.

CDAi = Maximum number of days available for crashing for activity i.

ΔDC = Extra Direct Cost due to crashing.

Rk = Maximum number of resources available. PUC = Penalty Unit Cost per day.

Ri = Resource assigned to activity i.

Ra = Sum of resources assigned to each activity on a particular day.

Framework Decision Variables

DT= Project Duration.

OWH= Overtime working Hours.

Ai = Start time of activity i.

Aj = Start time of succeeding activity j.

Xi = Activity i duration.

Xj = Succeeding activity j duration

Resource Constraints: The sum of resources assigned to each activity on a particular day should not be more than the maximum number of available resources.

$$R_a = \sum_{i=1}^{n} R_i \leq R_k \tag{1}$$

Activities Duration Constraints

Each activity has a minimum and maximum duration. The maximum duration corresponds to the normal duration while the minimum duration corresponds to the crash duration. Eq. (2) ensures that the activity duration remains within the allowed limits.

$$CD_p \le CDR_p \le ND_p \tag{2}$$

where,

$$CDR_p \ge 0 \text{ for all p}$$
$$CD_p \ge 0,\ CD_p \le ND_p \tag{3}$$
$$ND_p \ge 0$$

where,

$$CDR_q \ge 0 \text{ for all q}$$
$$ND_q \ge 0,\ \text{for all q} \tag{4}$$

Activities Relations Constraints

Each project activity is related to its successors through at least one of four relationship types, namely, finish to start (FS), start to start (SS), finish to finish (FF), or start to finish (SF). Considering activity i as the predecessor and j as the successor, these coefficients are FSij, SSij, FFij, SFij. During the forward pass, there is a need to calculate the early start and early finish of all activities on the network and during the backward pass, there is a need to calculate the late start and late finish of all activities on the network. The early start of first activity should be set to 0 as a start point of the network. Equations 5 to 8 explain the relations between the activities; FS, SS, SF, and FF, respectively:

Finish – to- start relationship (FS)

$$a_i + x_i + LT_{ij} \le a_j \tag{5}$$

Start - to - start relationship (SS):

$$a_i + LT_{ij} \le a_j \tag{6}$$

Start – to- finish relationship (SF):

$$a_i + LT_{ij} \le a_j + x_j \tag{7}$$

Finish – to - finish relationship (FF):

$$a_i + x_i + LT_{ij} \leq a_j + x_j \tag{8}$$

Equations 9 and 10 explain the forward pass rules while scheduling

$$ES_j = Max \left[(ES_i + FS_{ij}) \, ; \, (ES_i + SS_{ij}) \right] \tag{9}$$

$$EF_j = Max \left[(ES_j + X_j) \, ; \, (EF_i + FF_{ij}) \, ; \, (ES_i + SF_{ij}) \right] \tag{10}$$

Equations 11 and 12 explain the backward pass rules while scheduling:

$$LF_j = Min \left[(LS_i - FS_{ji}) \, ; \, (LF_i - FF_{ji}) \right] \tag{11}$$

$$LS_j = Min \left[(LF_j - X_j) \, ; \, (LS_i + SS_{ji}) \, ; \, (LF_i + SF_{ji}) \right] \tag{12}$$

Overtime Working Hour's Constraints

The total number of working hours allotted to a labor pre day as overtime should not violate the labor law requirements (A worker cannot be employed for more than 48 hours in a week. Weekly holiday is compulsory. Total working hours including overtime should not exceed 60 in a week so the maximum number of overtime working hours are 2 hour/day).

$$OWH_i = \frac{[(ND_i - CDR_i) \times R_i \times {NWH_i}/{day})]}{CDR_i \times R_i} \leq {2hr}/{day} \tag{13}$$

Objective Function

There is two objective functions. Each one is need to be solved in separate steps. First objective function is to minimize the resource requirement on a particular day up to the maximum available limit of resources and within the minimum possible project duration. MSP software is used to overcome this objectives.

$$\text{Minimize } R_a = \sum_{i=1}^{n} R_i \tag{14}$$

$$TD = \sum_{P=1}^{n} X_p + LT_{Pij} \tag{15}$$

The second objective function is to minimize the total project cost according to Equation (16). MS Excel Solver is used to overcome this objective.

$$TC = IDC + DC + \Delta DC + PNC + ORC \tag{16}$$

where,
Project Indirect Cost

$$ID = C_{OH} x D_T \tag{17}$$

Project Direct Cost

$$DC = \sum_{i=1}^{n} NC_i \tag{18}$$

Increase in direct cots due to crashing

$$\Delta DC = \sum_{i=1}^{n} CUC_i \ x(ND_i - CD_i) \tag{19}$$

Crash unit cost

$$CUC_i = \frac{CC_i - NC_i}{ND_i - CD_i} \tag{20}$$

Penalty Cost due to late completion

$$PNC = PND x PUC \tag{21}$$

Overtime Cost

$$ORC = \sum_{i=1}^{n} OWH_i \ x NW_i \ x OF \tag{22}$$

5. MODEL APPLICATION EXAMPLE

The nonlinear integer programming framework is able to find the best solution for the two cases of with and without considering overtime working as an activity expedition option. As explained earlier, to develop the framework, a two-step procedure is used. First, MSP software is used for resource leveling and then MS Excel solver is used to find the minimum project total cost satisfying both the resource and overtime working hour's constraints.

The model application example (Table 1) adopted from Isidor & Back 2012, and solved first cycle wise and then through the proposed NLIP framework for the both scenarios of TCT+CRS and TCT+CRS using overtime for activity expedition. Columns 8, 9 and 10 are added in order to use them in constraint resource scheduling, finding overtime working hours and cost, while columns 11 and 12 are added to perform the stochastic analysis. In table 1, the symbole "₹" stands for Indian Rupees (INR).

The indirect cost is assumed to be ₹ 90/day. The targeted deadline is 18 days and for each day, the project spends beyond the deadline (days) the contractor is panelized ₹ 100/day. The maximum number of resources available is assumed to be 8 resources per day and an

overtime factor of 0.5 is accepted by which the normal wages will increase during overtime working. Both the results (TCT+CRS and (TCT+CRS using overtime) are then compared through the remaining float for noncritical activates, Critical indexes of activities, Criticality ratio, and Probability of completion of the project on time. The proposed framework using overtime shows better performance in all those four conditions than the normal TCT+CRS. Fig. 1 shows the baseline schedule network and Fig. 2 shows the bar chart and resource graph.

The baseline schedule is loaded to Microsoft Project software and after applying resource leveling the resulted project duration is 25 days with ₹ 9,365 project total cost. Fig. 3 shows the baseline schedule network according to the resource leveled schedule and Fig. 4 show the bar chart and resource graph according to resource leveled schedule loaded to MSP. The network for baseline schedule prepared earlier (Fig. 2) modified according to the resource leveled schedule. The early start, early finish, late start, late finish and total float for each activity are recalculated. For application example one, to modify the network according to the resource levelled schedule, a finish to start relationship needs to be add between activity C and F. Because, according to the baseline schedule, activity F can start immediately after Start of the project, and there is no relation between activity F and C. While according to resource constraints, the start date of activity F has to be postponed until activity C also finish. Therefore, if we add a finish to start relationship between activity F and C due to recourse limitation, a different network will result with the same precedence relationships as baseline schedule with satisfying resource constraints. For this application example, four such relations are added between activities F-C, J-A, J-D and F-D. The resource constraints are applied through a start delay technique and identified through a dashed line in the activities network.

While applying TCT cycles to the resource leveled schedule, the optimum duration for normal TCT+CRS is found 16 days, and it is associated total cost is ₹ 8,270 satisfying resource constraints. And the optimum duration for TCT+CRS using overtime for activity expedition is 21 days with ₹ 8,980.3, Fig. 5, 6, 7, and 8 explain the cycles for normal TCT+CRS and Fig. 9, 10, 11, and 12 explain the cycles for TCT+CRS using overtime.

Table 1: Model Application Example, Available Data.

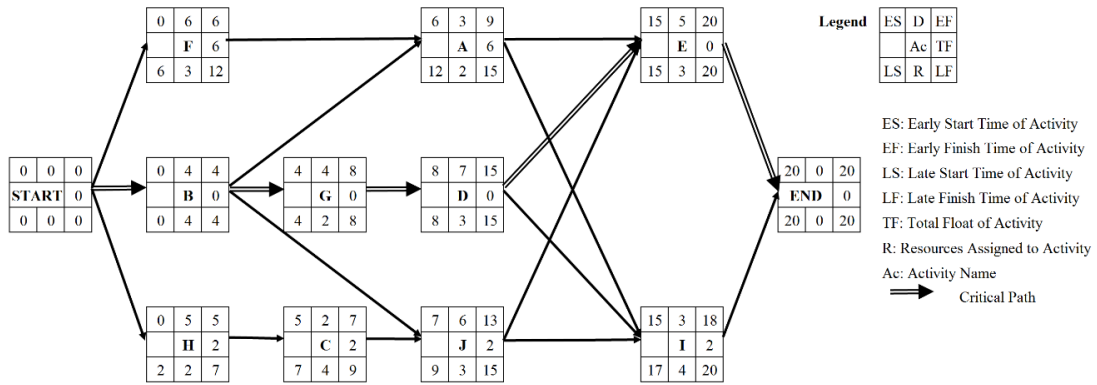| #	Activities	Normal duration (days)	Crash duration (days)	Normal Cost (₹)	Crash Cost (₹)	Cost Slope (₹)	Reso-urces	Normal working hours (hr/day)	Normal wages (₹/day)	Duration. Mean	Duration Std.Dev
1	A	3	2	400	430	30	2	8	50	3	1.2
2	B	4	2	1020	1100	40	4	8	60	4	3.14
3	C	2	2	350	350	0	4	8	40	2	1.8
4	D	7	5	1250	1300	25	3	8	50	7	1.25
5	E	5	3	825	895	35	3	8	65	5	1
6	F	6	3	610	700	30	3	8	55	6	2.3
7	G	4	2	430	490	30	2	8	70	4	2.5
8	H	5	2	525	660	45	2	8	45	5	1.5
9	I	3	2	390	410	20	4	8	50	3	0.5
10	J	6	3	615	765	50	3	8	60	6	1.5

Figure 1. Baseline schedule network diagram
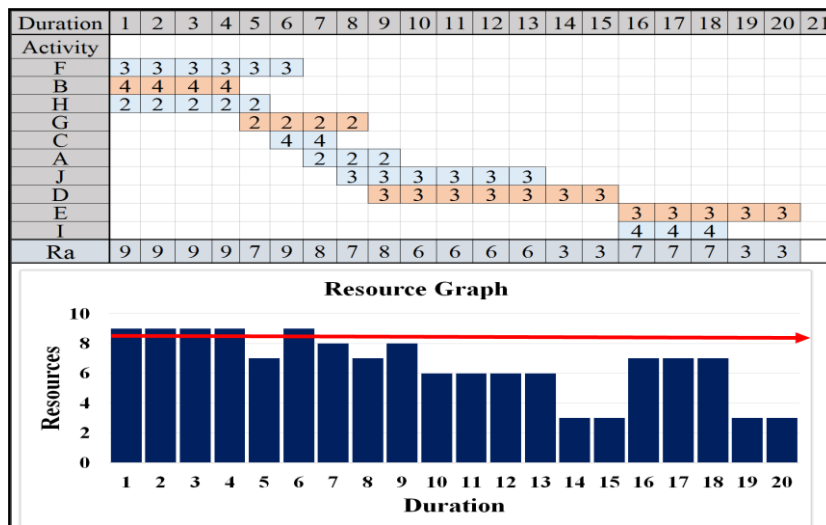
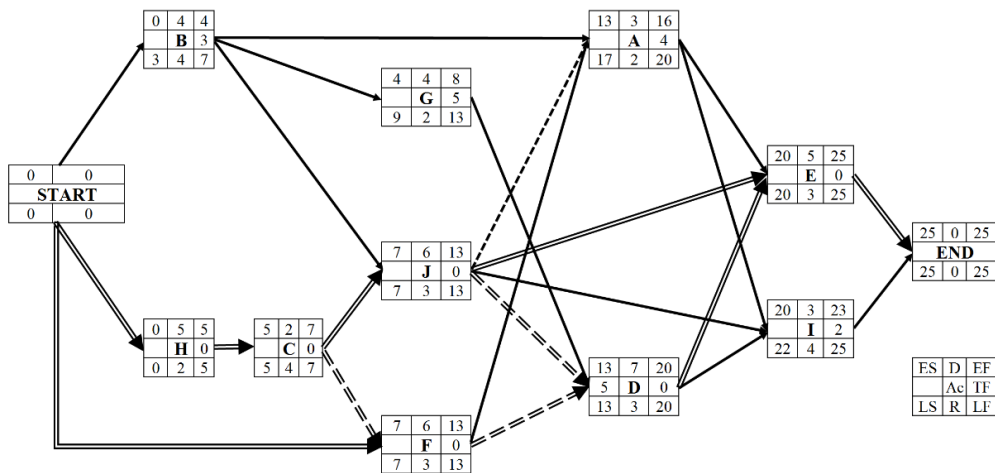Figure 2. Baseline Schedule Bar Chart and Resource Graph

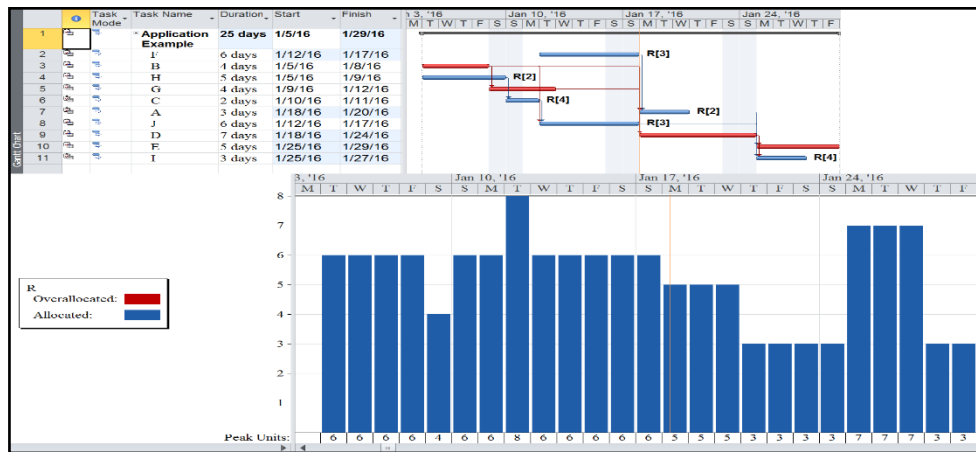Figure 3. Baseline schedule network according to resource levelled schedule

Figure 4. Baseline schedule bar chart and resource graph according to the resource levelled schedule
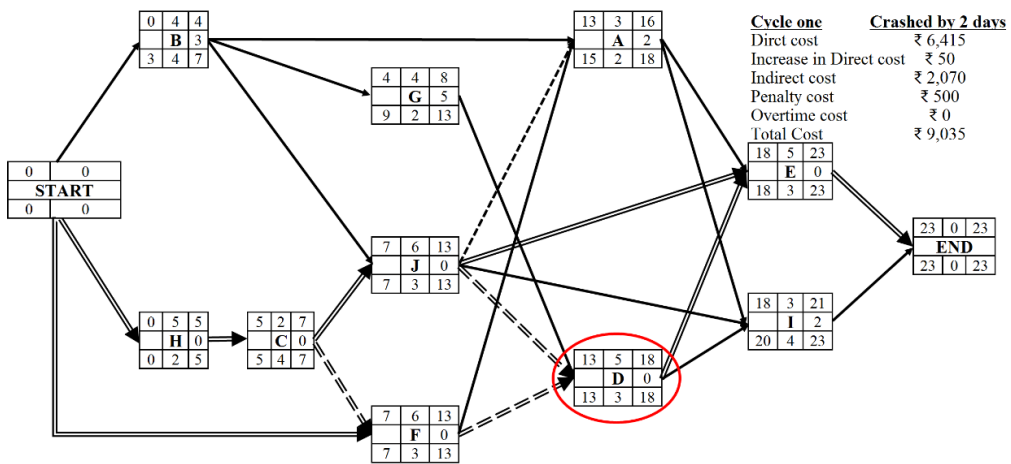

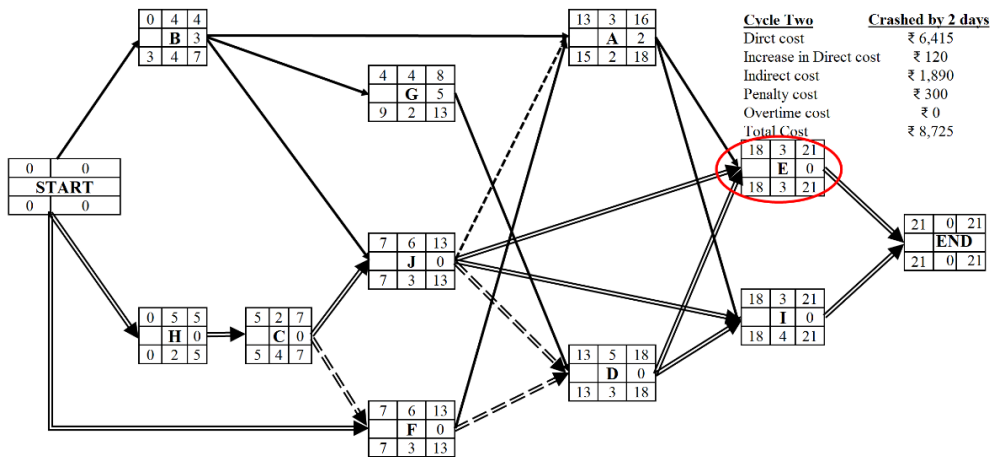Figure 5. Cycle one, normal TCT+CRS

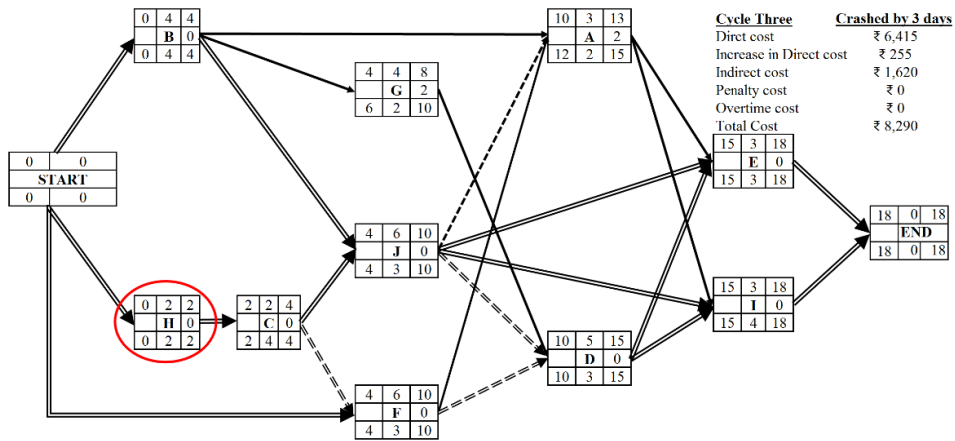
Figure 6. Cycle two, normal TCT+CRS
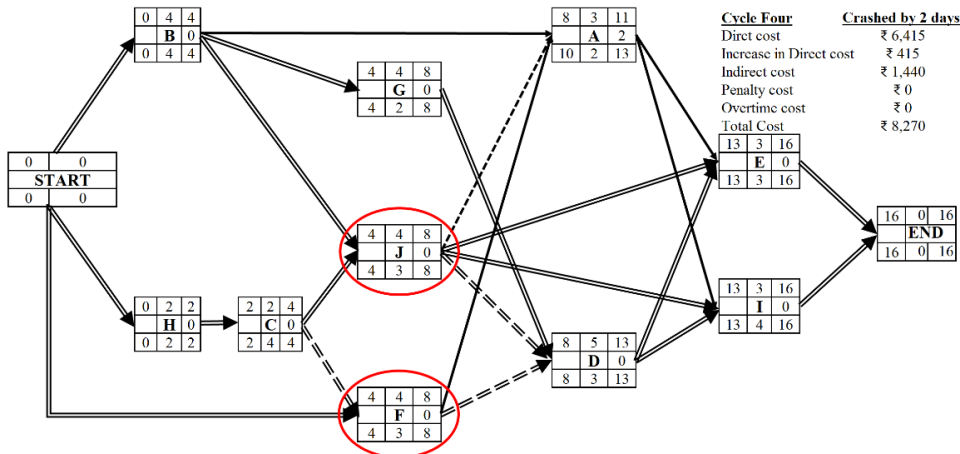
Figure 7. Cycle three, normal TCT+CRS

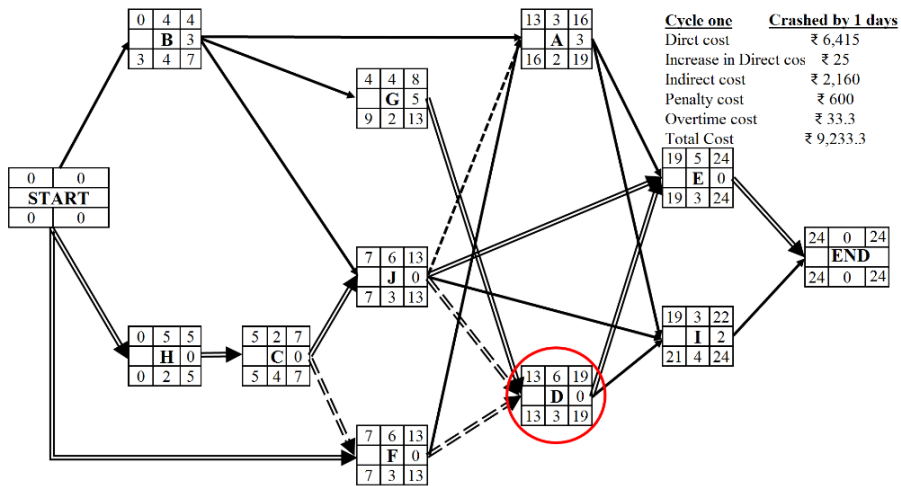Figure 8. Cycle four, normal TCT+CRS

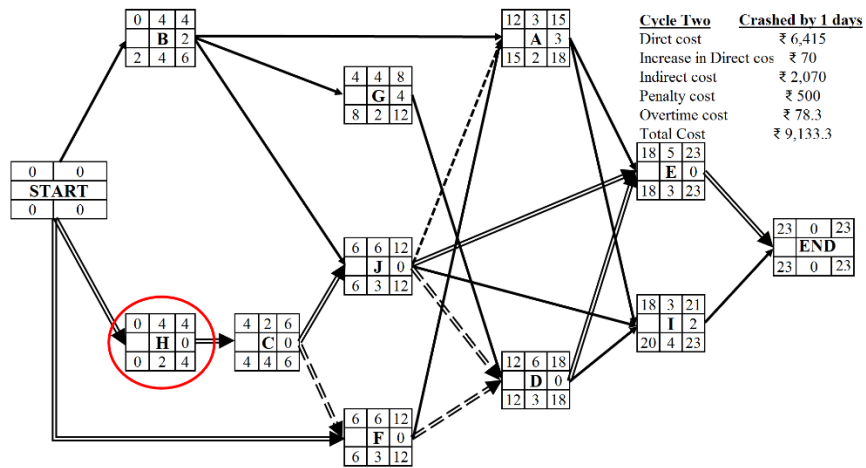Figure 9. Cycle one, TCT+CRS using overtime

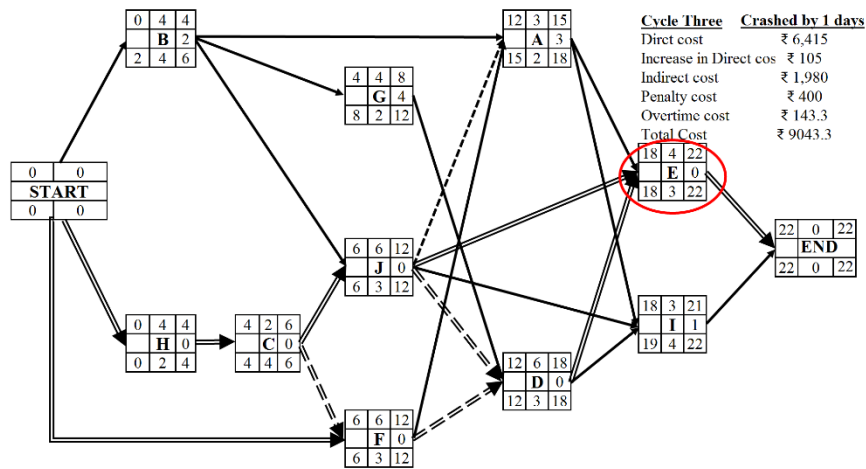Figure 10. Cycle two, TCT+CRS using overtime

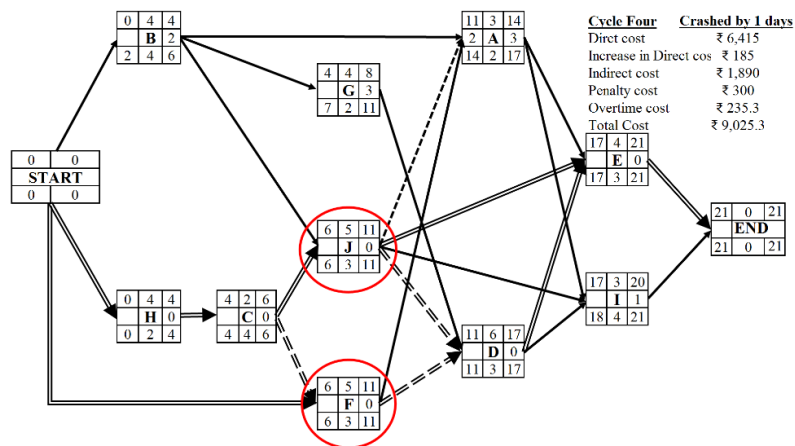

Figure 11. Cycle three, TCT+CRS using overtime

Figure 12. Cycle four, TCT+CRS using overtime

To apply the proposed NLIP framework to the same application example, a model table should be established in MS Excel that includes for each activity the required information with their corresponding formulas and links according to the equations stated above. Table 2 shows the model table.

Table 1: Model table for application example one

							Application Example, Model Table									
SN	Task Name	Normal Duration (days)	Crash Duration (days)	Normal Cost (₹)	Crash Cost (₹)	Cost slop (₹)	Max No. of days Available for crashing	No of Days Actually Crashed	Actual Increase in Direct Cost (₹)	Current Duration (days)	Resources Needs	Normal working hours/day	Overtime working hours/day	Normal Wages (₹) /Hour	Overtime Wages (₹)
1	Start	0	0	0	0	0	0	0	0	0	0	8	0	0	0
2	A	3	2	400	430	30	1	0	0	3	2	8	0	50	0
3	B	4	2	1020	1100	40	2	0	0	4	4	8	0	60	0
4	C	2	2	350	350	0	0	0	0	2	4	8	0	40	0
5	D	7	5	1250	1300	25	2	0	0	7	3	8	0	50	0
6	E	5	3	825	895	35	2	0	0	5	3	8	0	65	0
7	F	6	3	610	700	30	3	0	0	6	3	8	0	55	0
8	G	4	2	430	490	30	2	0	0	4	2	8	0	70	0
9	H	5	2	525	660	45	3	0	0	5	2	8	0	45	0
10	I	3	2	390	410	20	1	0	0	3	4	8	0	50	0
11	J	6	3	615	765	50	3	0	0	6	3	8	0	60	0
12	END	0	0	0	0	0.0	0	0	0	0	0	8	0	0	0

Define the upper mentioned objective functions for the both scenarios (Normal TCT+CRS and TCT+CRS using overtime for activity expedition) and set up the Excel solver window with identifying objective function, adjustable terms, and constraints. The constraints for the application example presented earlier are the minimum and maximum durations of the activities, the activities logic, and relations constraints, resource constraints, and overtime working hours constraints explained earlier according to equation 1 to 13 respectively. Table 3 shows the adjustable terms.

Table 2: Adjustable Terms

Activity	Number of days actually crashed	≤	Maximum Number of days available for crashing.	≥	Zero	=	Integer
Start	0	≤	0	≥	0	=	Integer
A	0	≤	1	≥	0	=	Integer
B	0	≤	2	≥	0	=	Integer
C	0	≤	0	≥	0	=	Integer
D	0	≤	2	≥	0	=	Integer
E	0	≤	2	≥	0	=	Integer
F	0	≤	3	≥	0	=	Integer
G	0	≤	2	≥	0	=	Integer
H	0	≤	3	≥	0	=	Integer
I	0	≤	1	≥	0	=	Integer
J	0	≤	3	≥	0	=	Integer
END	0	≤	0	≥	0	=	Integer

Run the model and check the solver report for the results.

6. ANALYSIS AND DISCUSSION OF THE RESULTS

After running the proposed nonlinear integer programming model and the developed framework for the application example, the following results are derived.

- The baseline schedule after applying resource leveling resulted with 25 days duration and 9,365 project total cost.
- The normal TCT+CRS resulted at 16 days optimum duration and ₹ 8,270 project total cost, while satisfying both the deadline and resource constraint.
- The optimum duration and it is associated cost for the same example, using overtime for activity expedition is 21 days and ₹ 9,025.3 respectively. Satisfying both overtime hour's constraints and resource constraints.

The resulted schedules in all three cases were compared then via criticality indexes, probability of completion of the project on time (using MCS, 10000 iterations), critical ratio, remaining float and flexibility of schedule. In all cases, the proposed model while using overtime for activity expedition shows better performance than the normal approach. Table 4 compares the remaining total float for the noncritical activities in all three cases (Baseline schedule, Normal TCT+CRS, and TCT+CRS using overtime). It can be noticed that the new proposed compression framework (Considering overtime working hour's constraints and using overtime for activity expedition is better in terms of remaining float as it finds an optimum solution that can save some total float for future use with a less risky cost.

Table 3: Remaining Float for Non-Critical Activities

Activity	Activities TF in days @25 days duration	Activity TF in days @21 Days Duration	Activity TF in days @16 Days Duration
A	4	3	2
B	3	2	0
C	0	0	0
D	0	0	0
E	0	0	0
F	0	0	0
G	5	3	0
H	0	0	0
I	2	1	0
J	0	0	0

In terms of the probability of finishing the project on time, the probability of finishing the project within 25 days is found to be 30.36%, with a mean of 26.591 and standard deviation of 3.094 after 10000 trails of MCS using @Risk. The probability of finishing the project within 21 days when overtime is used for activities expedition is 20.18%, with a mean of 25.627 and standard deviation of 5.533 after 10000 trails of MCS through @Risk. While the probability of finishing the project within 16 days, normal TCT+CRS is 13.42% with a mean 19.43 and standard deviation 3.098. From the previous probabilities found, it can be seen that when overtime used for activity crashing, the probability of finishing the project is higher than the normal case.

The optimum solution found (without Considering overtime), in comparison with the

optimum solution considering the overtime constraints and expediting activities through overtime, is also slightly better in terms of activities' criticality indices that are found using Monte Carlo Simulation through @Risk (Add-in in Excel) and presented in Table 5.

The criticality ratio is calculated as a ratio between the numbers of critical activities to the total number of activities. The criticality ratio in the normal case (resource leveled schedule) scenario is 0.6 while TCT+CRS resulted in criticality ratio of 0.9 and constraint resource scheduling using overtime for activity expedition resulted in a criticality ratio of 0.6. It shows that the schedule criticality is less when overtime is used for activity expedition. Fig. 13 compares the results between the optimum solutions found for normal TCT+CRS without considering overtime for activity expedition and the optimum solution found while using overtime for activity crashing for the purpose of TCT in CRS.

Table 4: Activities criticality indices

Activity	Activity Critical Index, Baseline Schedule (25 Days Duration)	Activity Critical Index, TCT+CRS (16 Days Duration)	Activity Critical Index, TCT+CRS Using Overtime (21 Days Duration)
Start	100%	100%	100%
A	0.59%	5.24%	2.28%
B	16.77%	48.24%	26.35%
C	83.29%	51.90%	73.75%
D	99.43%	94.84%	93.73%
E	96.45%	49.68%	81.26%
F	28.16%	40.38%	31.68%
G	5.89%	29.87%	13.51%
H	83.29%	51.90%	73.75%
I	3.55%	50.32%	18.74%
J	50.03%	38.93%	45.98%
End	100%	100%	100%

From Fig. 13, it can be seen that the project total cost and duration in case of using overtime working for crashing and expediting activities, is higher than the project total cost and duration in case of normal TCT+CRS. The increase in the project total cost and duration while using overtime for crashing of activities is related to the increase in the direct cost that accounts for the overtime wages, penalty cost for days beyond the deadline (3 days) and the indirect cost due to longer duration camper to the normal TCT+CRS case. But the total direct cost is decreasing when to consider overtime hour's constraints due to less crashing of critical activities. Although the proposed framework presents a curve with a higher cost, and duration, the difference between project total costs in both scenarios are equal to $ 755.3. Table 6 explain this matte.
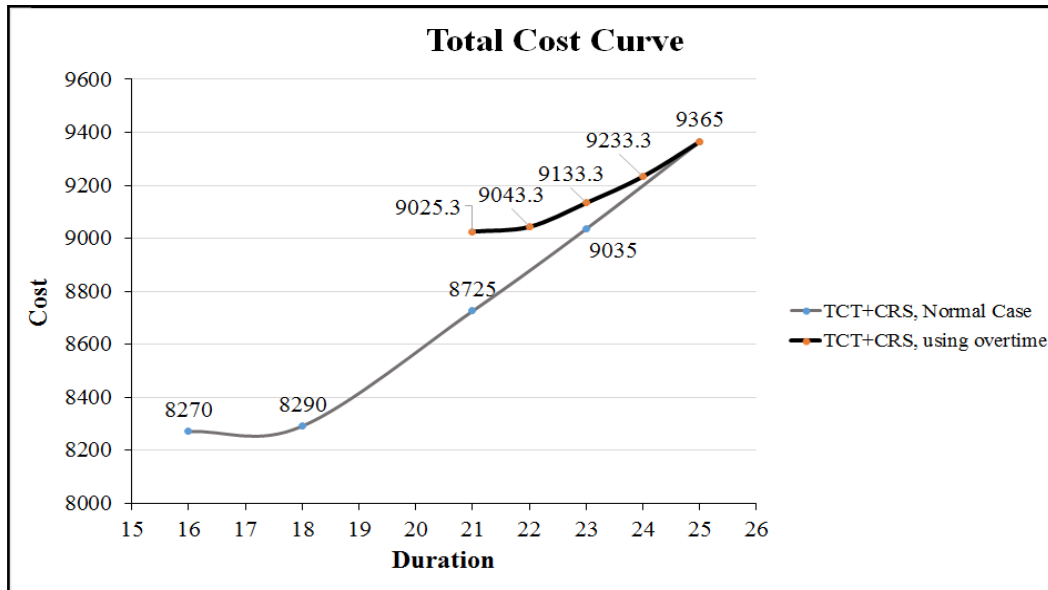
Figure 13. Result Comparison between Normal TCT+CRS and TCT+CRS using Overtime

Table 5: Cost Difference

Cost	Considering Overtime	Without Considering Overtime
Direct cost	₹ 6415	₹ 6415
Increase in direct cost	₹ 185	₹ 415
Indirect cost	₹ 1890	₹ 1440
Penalty	₹ 300	0
Overtime cost	₹ 235.3	0
Total	₹ 9,025.3	₹ 8270
Difference	₹ 755.3	

In this example, this higher cost paid accounts and quantifies the important of overtime working used for activities crashing in case of limited resource availability, and can save amount associated with risks appearing from project flexibility loss and crushing activity without adding more resources to that. Decision makers or project managers, depending on the nature of their projects, are free to choose between the two curves, whether to stick to the normal compression method and bear the risk associated with losing total float, low probability of completion and more tight schedule or use the new curve and be at the safe side while maintaining a compressed schedule in resource constraints conditions.

The framework is validated through five different examples solved using the proposed nonlinear integer frameworks for time-cost trade-off and constraint resource scheduling in both cases. First, normal case, without considering overtime, and then using overtime as activity expedition option following by comparison between both resulted schedules in

terms of probability of finishing on time. Table 7 summarizes the results of the five examples found via the non-linear integer programming taking help of MSP and MS Excel solver along with their probability of finishing the project on time for all three cases (Normal CRS, TCT+CRS, and TCT+CRS using overtime).

Table 6: POF summary of the five example results

Example	Normal CRS			TCT+CRS			TCT+CRS using Overtime		
	Project Duration	Total Project Cost	POF %	Project Duration	Total Project Cost	POF %	Project Duration	Total Project Cost	POF %
Example 1 (Sanjay Tiwari)	12	4800	31.11	10	4075	29.14	11	4527.8	30.81
Example 2 (Tarek Hegazy)	18	11000	50	10	10000	49.22	16	10727	50.0
Example 3 (Rana El Haj))	28	14340	24.64	23	13415	17.76	25	13855	22.98
Example 4 (Elbeltagi)	64	47300	44.47	55	45075	25.84	55	45392	26.80
Example 5 (M. Geda)	22	84800	47.11	16	76900	43.29	21	82933	47.17

In light of the results presented in Table 7, one can notice the improved probability of finishing the project on time if overtime is using for activity crashing in case of limited resources availability. The NLIP framework is used to find an optimum/efficient solution to the optimization problem where we cannot add an extra resource to any activity for crashing purpose due to resource constraints. And instead, increasing working hours or overtime working is used to expedite activities in order to meet the minimum project total cost within the possible minimum duration, while satisfying resources and overtime working hour's constraints. The developed frameworks indeed allow the decision makers to experience a new tradeoff between time-cost in resource constraint condition while improving the chances of meeting the targeted project duration within the planned budgeted cost.

7. CONCLUSION

In most of the construction projects, the deadline and resource limits are two in action constraints that need to be fulfilled together. This paper focuses on an effective method for considering TCT+CRS simultaneously while using overtime for activity expedition. Various problems are solved by the researcher for this purpose, However, most of the existing methods didn't consider the fact that up to what extent we can crash an activity in resource constrained condition and what is the most possible and practical option for acceleration of activities and project where limited resources are available. So, in the case of resource constraints a simple case arises in the scheduling of overtime work by using of weekend or evening time, the completion time for an activity as measured in calendar days will be

reduced. However, the cost may increase due to overtime wages of overtime work, also the chance of accidents and quality problems are higher during overtime working period and there is a possibility for rework which must be corrected. So, again costs may increase, and the number of working hours as overtime implied to each labor should not violate the labor law. Therefore, this paper focused on an NLIP framework introducing overtime working hours into the time-cost trade-off in constraint resource condition. A two-step procedure is developed to achieve the result, using Excel solver and MSP. The proposed framework avoid repeatedly applying the resources to the various feasible schedules as explained by Sanjay Tiwari [6]. Therefore, the proposed NLIP framework required less time to generate the optimum solution and limit the TCT and CRS to two separate steps only. On the other hand, considering overtime for activity expedition improve the probability of completion of the project on time, enhance the flexibility of the schedule and decrease the risk associated with the criticality of schedule, delay and completion on time compare to the normal TCT+CRS.

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