OPTIMAL DESIGN OF COLUMNS FOR AN INTERMEDIATE MOMENT FRAME UNDER UNIAXIAL MOMENT AND AXIAL LOADS

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ABSTRACT

The present study addresses optimal design of reinforced concrete (RC) columns based on equivalent equations considering deformability regulations of ACI318-14 under axial force and uniaxial bending moment. This study contrary to common approaches working with trial and error approach in design, at first presents an exact solution for intensity of longitudinal reinforcement in column section by solving equivalent equation. Then, longitudinal and transverse reinforcement details are assessed regarding the previous step results and where achieving the lowest steel consumption design in the column is selected as the optimum. In addition to optimizing column cross-section dimension by implementing single-variable optimization methods, the effect of axial force, bending moment and concrete compressive strength variations on the column cross-section dimension, intensity of longitudinal reinforcement, construction costs and total weight of consumption steel have been investigated. The investigation on the validity of the proposed method was assessed and signified through comparison with the existed work in the literature. Finding an exact solution considering all regulations and constraints is the advantage of this method in determining optimized RC column.

1. INTRODUCTION

A set of assumptions are generally taken into account in concrete structural design. Design codes such as ACI318, have common design assumptions for concrete members similar to beams, columns and RC slabs, but follow different design approaches [1]. As RC beams just

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carry bending moments, the intensity of steel required for reinforcing concrete is easily calculated, implementing algebraic equations [2]. But in RC columns design, the simultaneous effect of subjected axial load and bending moment should be taken into account. Simultaneous effect of axial load and bending moment makes different design conditions and presents different methods to find the required area of longitudinal reinforcement. P-M interaction curves were considered as a powerful tool in columns design after being introduced by Whitney and Cohen in 1956 [3]. This methodology is based on the solution of equilibrium equations over the column cross section with specific reinforcement details [4]. So, the optimum design is when the point illustrating axial load and bending moment over the cross-section is located inside the zone surrounded by M=0, P=0 axes and interaction curve [5]. Introducing P-M interaction curves has made design of RC columns much easier [6]. However, common methods used for designing columns require repetitive procedures as trial and error to find suitable reinforcement over a column section under applied loads [7].

Repetitive procedures can find responses meeting all limits, but complexity of design conditions for a reinforced concrete column with a large number of responses makes determination of the optimal response highly time and cost consuming [8]. Optimization methods based on gradient and metaheuristic algorithms [9-12] are usage applications to find the optimum design in RC structures. [13-17]. Adamu and Karihaloo [18] optimized columns subjected to uniaxial bending, reinforced in parallel sides based on DCOC, a suggested method implementing Lagrngian equations. De Medeiros and Kripka [19] optimized a column cross-section and its longitudinal reinforcement based on metaheuristic harmony search algorithm under the effects of environmental parameters They investigated section strength under loading with the help of interaction curve. Sánchez-Olivares and Tomás [20] optimized column cross-section and longitudinal reinforcement under biaxial bending using optimized firefly metaheuristic algorithm. Gharehbaghi and Fadaee [21] implemented PSO algorithm to optimize column longitudinal rebar dimensions and cross-section in reinforced concrete frame under seismic loads. Also, Gholizadeh and Aligholizadeh [22] and Gheyratmand et al. [23] optimized columns in RC frame by metaheuristic algorithms and based on a database for column sections and longitudinal reinforcement.

Contrary to metaheuristic algorithms, some of the studies have been looking for an exact solution for longitudinal reinforcement cross-section of every column section using assumptions and equations. Aschheim et al. [2] studied optimization of intensity of longitudinal reinforcement of RC columns subjected to uniaxial bending based on European concrete standard and by creating domains based on specific points in interaction curve. Lee et al. [7] optimized longitudinal reinforcement for columns with plane reinforcement in parallel sides, axial bending and different load combinations using Reinforcement Sizing *Diagrams* (RSD) and mathematical methodologies. Aschheim et al. [24] solved equilibrium equations for a column with rectangular section to optimize longitudinal reinforcement in a section with discrete reinforcement in the cross section using discrete values for diameter of rebar under biaxial bending. Aschheim et al. [1] carried similar studies but in 2 or 4 sides of the column section, using sequential search nonlinear methods. In the latter research, required rebar discretization is conducted based on stress distribution in each side of column section, beam or concrete walls.

In present study, required intensity of longitudinal reinforcement in square section under uniaxial bending and axial load is determined by creating domains based on sectioning the height of neutral axis in column section. In this method, an exact solution is obtained by solving equivalent equations based on mathematical theory, for the necessary reinforcement area for each section of column. As most columns are utilized with axisymmetric reinforcement in all four sides in construction, reinforcement is considered as steel plates with constant thickness and discretization is carried based on a list catalog. All possible states of longitudinal and transverse reinforcement were accounted for as in a catalog list, regarding all intermediate deformability conditions and regulations for columns, and the one with least amount of consumption steel in column is selected as optimum choice for column.

In this study, the optimization problem is divided into two parts for optimizing column dimension based on the lowest construction cost and optimizing the pattern of longitudinal and transverse rebar in a specific dimension. The square column dimension is the only parameter for optimization problem. Single variable optimization algorithm of Golden Section Search Method [25] is implemented to find the best response for column dimension under uniaxial bending and axial load. Target function is composed of concrete, steel and formwork costs. Steel costs in target function includes longitudinal and transverse rebar in the column. Effects of axial load, bending moment and concrete characteristic compressive strength are investigated based on column design parameters such as column dimension, longitudinal rebar cross-section, construction cost and total weight of consumed steel.

2. DESIGN ASSUMPTIONS

Design of the column cross-section is conducted based on ACI318−14 code. Assumptions are indicated in a free body diagram as shown in Fig. 1. In case of a compressive force, it is presumed positive and the applied moment is positive if cross-section's upper layer is under compression. Also, steel plates are utilized as reinforcement in column cross-section as in Fig. 1, with a constant thickness t in all four sides.
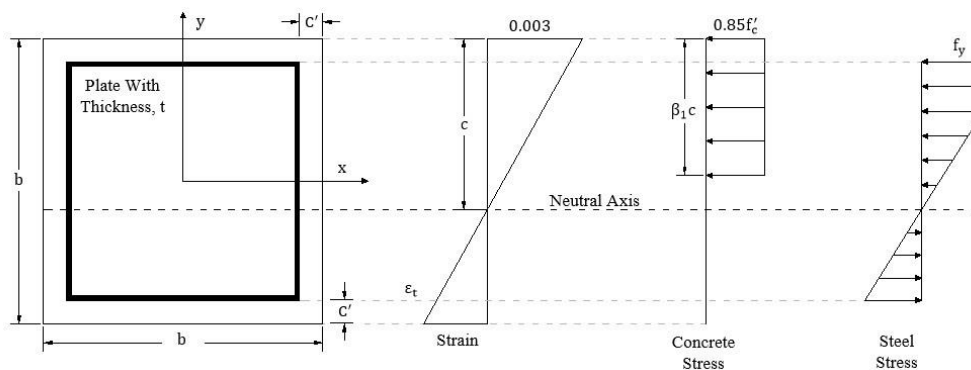

Figure 1. Free body diagram of column cross-section

In Fig. 1, b is the column breadth, c' is the concrete cover depth in two directions, ε_t is the strain in the farthest tensile row in column cross-section, c is neutral axis height, f_c' is the concrete compressive strength and β_1 is the height coefficient of equivalent rectangular

compressive stress block over the section. According to ACI (22-2-2-4), to estimate real concrete compressive stress distribution in final failure moment properly, stress distribution is assumed as Whitney rectangular stress distribution of $0.85f_c'$ intensity[3]. The height of rectangular compressive stress block is calculated as $a = \beta_1 c$. β_1, and height coefficient of equivalent rectangular compressive stress block is calculated based on Equation (1).

$$\begin{array}{ll} \beta_1 = 0.85 & 17 \leq f_c'(MPa) \leq 28 \\ \beta_1 = 0.85 - 0.00714(f_c' - 28) & 28 \leq f_c'(MPa) < 55 \\ \beta_1 = 0.65 & f_c'(MPa) \geq 55 \end{array} \qquad (1)$$

To calculate axial load and corresponding moment, steel behavior is assumed elastoplastic. So tensile or compressive stress in steel is calculated by multiplying elasticity modulus by steel strain in elastic zone and stress in steel rebar is equal to yield stress in plastic zone. Strain is assumed positive when it is compressive and vice versa. With this approach the best possible states for the consumed steel stress and strain are obtained and the number of unknowns demotes.

As cross-section is subjected to axial load and uniaxial bending, steel plates in side sections will have a totally different conditions comparing to upper and lower sides. Under predefined loading conditions as indicated in Fig. 1. some parts of lateral sides may act in elastic zone and other parts in plastic. Therefore, calculating strain in any height of the section can determine elastic or plastic behavior of that part. According to strength-based design mentioned in ACI318-14, equilibrium conditions must be applied in each section. C_c, C_s and T_s are the loads carried by concrete, compressive steel and tensile steel, respectively. C_c is calculated from Equation (2):

$$\begin{array}{llll} if \; c > {}^b/_{\beta_1} \; ; & a = b \; , & C_c = 0.85f_c'b^2 \\ otherwise \quad ; & a = \beta_1 c \; , & C_c = 0.85f_c'ab \end{array} \qquad (2)$$

If axial load is much larger than moment, neutral axis height increases. If it goes up more than ${}^b/_{\beta_1}$, all concrete will be subjected to compressive stress and height of Whitney rectangular stress block will be limited to *b*.

Compressive and tensile forces in upper and lower planes are calculated by multiplying stress in each part of the steel plate by its corresponding cross-section. To simplify and reduce the equilibrium equations, reduction in concrete cross-section under compressive stress has been ignored. Because steel plates have very negligible thickness, this will not have any effect on final results. Stress in upper and lower planes are calculated by Equation (3):

$$\begin{array}{llll} C_{s,i} = f_{s,i}'A_{s,i} \; ; & A_{s,i} = t(b - 2d') \; , & i = Top \; or \; Bottom \; Plate \\ T_{s,i} = f_{s,i}A_{s,i} \; ; & A_{s,i} = t(b - 2d') \; , & i = Top \; or \; Bottom \; Plate \end{array} \qquad (3)$$

A_s is area of steel plate cross-section, f_s is steel plate tensile stress, f_s' is steel plate compressive stress, *t* is steel plate thickness, *b* is section width and *d'* is the distance of upper steel plate center from the farthest tensile row. According to ACI (22-2-2-1), maximum strain in the farthest tensile row is assumed *0.003*. Using Bernoulli principle assuming that planes keep their plate shape after deformation and considering strain in the farthest tensile

row, strain is determined in every height of the section. So, f_s and f_s' are determined from Equations (4) and (5) assuming elastoplastic behavior for steel.

$$f_s' = E_s\varepsilon_s' = E_s(0.003(1 - h_i/c)) \qquad\qquad If \qquad \varepsilon_s' \le f_y/E_s$$
$$f_s' = f_y \qquad\qquad\qquad Otherwise \tag{4}$$
$$f_s = E_s\varepsilon_s = E_s(0.003(h_i/c - 1)) \qquad\qquad If \qquad \varepsilon_s \le f_y/E_s$$
$$f_s = f_y \qquad\qquad\qquad Otherwise \tag{5}$$

where f_y is the steel yield stress, E_s is steel elasticity modulus, ε_s' is compressive strain, ε_s is tensile strain and h_i is the steel section height from the farthest tensile row. For steel plates in lateral sides of the reinforced concrete column section, axial load and bending moment are calculated by integrating part of the area under stress-moment curve resulted from stress around section's plastic axis. As section is symmetric, plastic axis height is $b/2$. Based on equilibrium equations, axial load and bending moment are functions of strength reduction factor $\emptyset$, steel plate thickness t and neutral axis height c. In this state the number of unknowns is more than that of equilibrium equations and problem has one degree of freedom. Equilibrium equations are stated as Equations (6) and (7).

$$P_u/\emptyset = P_n = \sum_{internal} P(t,c,\emptyset) = 0.85f_c'ab + \int f_s dA_s + \int f_s' dA_s' \tag{6}$$

$$M_u/\emptyset = M_n = \sum_{internal} M(t,c,\emptyset) = 0.85f_c'ab\left(b/2 - a/2\right) + \int f_s y dA_s + \int f_s' y dA_s' \tag{7}$$

where P_u and M_u are axial load and resulting uniaxial bending moment of different load combinations on the cross-section, respectively. Also P_n and M_n are nominal axial load and bending moment subjected to the section, respectively, calculated by dividing the values obtained from analysis by defined reduction factors from standards.

In order to minimize the unknowns and implementing algebraic methods, equilibrium equations should be solved by determining different domains for neutral axis height in the column cross-section. If neutral axis height is in allowable limit of the domain, it is selected as an acceptable response. According to ACI (21-2-2) and considering strain in the farthest tensile row over the section, strength reduction factor $\emptyset$ is variable and so the section can be compression-controlled, tension-controlled and in transition zone. Domains are classified based on the neutral axis height and strength reduction factor $\emptyset$. Application of strength reduction factor in classification of domains has demoted the number of unknowns and reduced the number of degree of freedoms to zero. Therefore, the problem is easily solved by algebraic methods. This classification can be easily calculated and used for different yield strengths. Table1 indicates the neutral axis height domain for yield strengths of 400-420 MPa.

In neutral axis height domain, the section is under large axial load and low bending moments in domain No.1. Then, axial load diminishes and uniaxial bending moment promotes. Also the section enters transition zone from compression-controlled and finally enters tension-controlled zone in domains No.1 to 11. Strength reduction factors in each domain also are shown in Table 1. Based on the effective dimensions in the section and the

breadth of the column, different conditions can come up in upper and lower limits of neutral axis height which are also shown in Table 1.

Table 1: Neutral axis division for steel with yield strength in the range of 400-420 MPa

Domain No.	Situation	Lower Bound of Neutral Axis	Upper Bound of Neutral Axis	$\emptyset$
1	--------	$E_s\varepsilon_{cu}d / (E_s\varepsilon_{cu} - f_y)$	--------	0.65
2	--------	$E_s\varepsilon_{cu}b / 2(E_s\varepsilon_{cu} - f_y)$	$E_s\varepsilon_{cu}d / (E_s\varepsilon_{cu} - f_y)$	0.65
3	--------	b / β_1	$E_s\varepsilon_{cu}b / 2(E_s\varepsilon_{cu} - f_y)$	0.65
4	--------	d	b / β_1	0.65
5	$b < A^*$	$E_s\varepsilon_{cu}d' / (E_s\varepsilon_{cu} - f_y)$	d	0.65
	$b \geq A^*$	$E_s\varepsilon_{cu}d / (E_s\varepsilon_{cu} + f_y)$	d	0.65
6	$b < A^*$	$b / 2$	$E_s\varepsilon_{cu}d' / (E_s\varepsilon_{cu} - f_y)$	0.65
	$b \geq A^*$	$b / 2$	$E_s\varepsilon_{cu}d / (E_s\varepsilon_{cu} + f_y)$	$0.233 + 0.25(d/c)$
7	$b < A^*$	$E_s\varepsilon_{cu}d / (E_s\varepsilon_{cu} + f_y)$	$b / 2$	0.65
	$b \geq A^*$ & $b < B^*$ & $d < C^*$	$E_s\varepsilon_{cu}d' / (E_s\varepsilon_{cu} - f_y)$	$b / 2$	$0.233 + 0.25(d/c)$
	$b \geq A^*$ & $b < B^*$ & $d \geq C^*$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	$b / 2$	$0.233 + 0.25(d/c)$
	$b \geq A^*$ & $b \geq B^*$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	$b / 2$	$0.233 + 0.25(d/c)$
8	$b < A^*$ & $b > D^*$	$E_s\varepsilon_{cu}b / 2(E_s\varepsilon_{cu} + f_y)$	$E_s\varepsilon_{cu}d / (E_s\varepsilon_{cu} + f_y)$	$0.233 + 0.25(d/c)$
	$b < A^*$ & $b \leq D^*$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	$E_s\varepsilon_{cu}d / (E_s\varepsilon_{cu} + f_y)$	$0.233 + 0.25(d/c)$
	$b \geq A^*$ & $b < B^*$ & $d < C^*$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	$E_s\varepsilon_{cu}d' / (E_s\varepsilon_{cu} - f_y)$	$0.233 + 0.25(d/c)$
	$b \geq A^*$ & $b < B^*$ & $d \geq C^*$	$E_s\varepsilon_{cu}d' / (E_s\varepsilon_{cu} - f_y)$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	$0.233 + 0.25(d/c)$
	$b \geq A^*$ & $b \geq B^*$	$E_s\varepsilon_{cu}b / 2(E_s\varepsilon_{cu} + f_y)$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	0.9
9	$b < A^*$ & $b > D^*$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	$E_s\varepsilon_{cu}b / 2(E_s\varepsilon_{cu} + f_y)$	$0.233 + 0.25(d/c)$
	$b < A^*$ & $b \leq D^*$	$E_s\varepsilon_{cu}b / 2(E_s\varepsilon_{cu} + f_y)$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	0.9
	$b \geq A^*$ & $b < B^*$ & $d < C^*$	$E_s\varepsilon_{cu}b / 2(E_s\varepsilon_{cu} + f_y)$	$\varepsilon_{cu}d / (\varepsilon_{cu} + \varepsilon_y)$	0.9

$b \geq A^*$ & $b < B^*$ & $d \geq C^*$	$E_s\varepsilon_{cu}b \big/ 2(E_s\varepsilon_{cu} + f_y)$	$E_s\varepsilon_{cu}d' \big/ (E_s\varepsilon_{cu} - f_y)$	0.9
$b \geq A^*$ & $b \geq B^*$	$E_s\varepsilon_{cu}d' \big/ (E_s\varepsilon_{cu} - f_y)$	$E_s\varepsilon_{cu}b \big/ 2(E_s\varepsilon_{cu} + f_y)$	0.9
10	--------	d'	$Lower\ Bound_{Domain\,9}$	0.9
11	--------	0	d'	0.9

$$A^* = 2\left(E_s\varepsilon_{cu}d' \big/ (E_s\varepsilon_{cu} - f_y)\right) \qquad B^* = 2d'\left((E_s\varepsilon_{cu} + f_y) \big/ (E_s\varepsilon_{cu} - f_y)\right)$$

$$C^* = (8/3)\left(E_s\varepsilon_{cu}d' \big/ (E_s\varepsilon_{cu} - f_y)\right) \qquad D^* = (3/4)d(1 + (f_y \big/ E_s\varepsilon_{cu}))$$

where d is the neutral axis height, and ε_{cu} is the maximum strain in concrete. It should be mentioned that if we face a set of possible responses in different domains, the one with the least steel consumed for longitudinal rebar is selected.

3. DISCRETIZATION OF DESIGN STEEL

In order to provide executable responses from solving equations given in the previous steps, discretization process must take place. This is done by discretizing longitudinal rebar. Fig. 2 Presents the longitudinal and transverse rebar discretization.
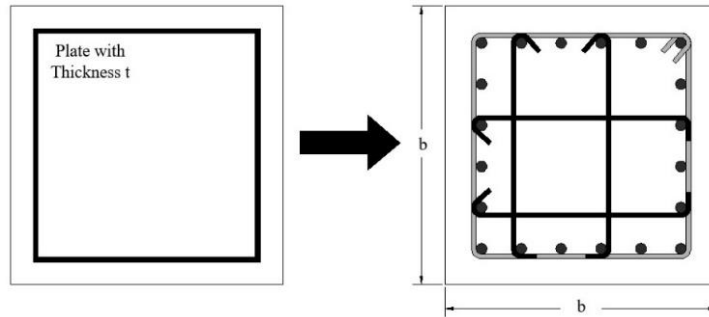

Figure 2. Longitudinal and transverse rebar discretization

3.1 Longitudinal reinforcement

A set of possible states based on the number of longitudinal reinforcement with specific diameter which are consistent with the section dimensions are generated. These states are generated based on net spacing between longitudinal rebar and if they meet the requirements, they will be introduced as potential responses.

3.1.1 Longitudinal reinforcement constraints

1. According to ACI (10-6-1-1), longitudinal reinforcement cross-section in column must have a minimum value of $0.01A_g$ and a maximum value of $0.08A_g$ where A_g is the gross area of column cross-section. Longitudinal reinforcement placement should have larger area than the one calculated in previous step.

2. According to ACI (25-2-3), minimum spacing between longitudinal rebar is calculated as follows:

$$S_{min} = \{1.5d_b, 40\ mm, (^4/_3)d_{agg}\}$$ (8)

where S_{min} is the minimum net spacing of longitudinal rebar, d_b is the longitudinal rebar diameter and d_{agg} is the largest concrete aggregate diameter.

3. According to ACI (10-7-3-1), minimum number of longitudinal reinforcement in a column with rectangular cross-section is 4.

4. To investigate all possible states of longitudinal reinforcement, longitudinal rebar diameters are assumed:

Longitudinal Rebar Dimension (mm): 12, 14, 16, 18, 19, 20, 22, 24, 25, 26, 28, 30, 32, 34, 36, 38, 40

3.2 Transverse rebar

As intermediate formability is addressed, related constraints should be taken into account in determination of transverse rebar. Nominal shear strength of the reinforced concrete beam is defined based on ACI (22-5-1-1).

$$V_n = (V_c + V_s)$$ (9)

where V_n is nominal shear strength of section, V_c is concrete nominal shear strength, V_s is transverse rebar nominal shear strength. According to ACI (22-5-6-1), Equation (10) is used to determine concrete nominal shear strength in this study.

$$V_c = 0.17(1 + \frac{P_u}{14A_g})\sqrt{f_c'}bd$$ (10)

Nominal shear strength of standing transverse rebar is calculated based on ACI (22-5-10-5-3) by using Equation (11).

$$V_s = \frac{A_v f_{yt} d}{S_v}$$ (11)

In Equation (11), A_v is transverse rebar cross-section, f_{yt} is transverse rebar yield strength and S_v is center-to-center distance of transverse rebar in column central area. As column design with intermediate formability is addressed, according to ACI (18-4-3-1) design shear force should equate the minimum value of two following limits:
1. The shear associated with development of nominal moment strengths of the column at each restrained end of the unsupported length due to reverse curvature bending.
2. Maximum shear obtained from factored load combinations including earthquake where $\Omega_0 E$ is replaced with E. Ω_0 is over strength factor which is 3 for intermediate frames.

3.2.1 Transverse reinforcement constraints

1. According to ACI (10-5-1-1), design shear strength should be larger than calculated shear strength in that point.

$$\emptyset V_n \geq V_u \tag{12}$$

where V_n is nominal shear strength, V_u is analyzed shear strength and $\emptyset$ is shear strength reduction factor equal to 0.75 based on ACI (21-2).

2. According to ACI (18-4-3-3), transverse rebar should be placed along the length L_0 with a spacing of S_0.

$$S_{0,max} = min\left\{ \left(b/2 \right), 8\Phi_L, 24\Phi_T, 300\ mm, S_v \right\} \tag{13}$$

$$L_{0,max} = min\left\{ b, L_n/6, 450\ mm \right\} \tag{14}$$

$S_{0,max}$ is the maximum spacing of transverse rebar in length L_0, Φ_L is the longitudinal rebar diameter, Φ_T is transverse rebar diameter, L_n is column net height and $L_{0,max}$ is the maximum allowable length L_0 in column upper and lower surfaces.

3. According to ACI (18-4-3-4), the first transverse rebar spacing should not exceed $S_0/2$.

4. According to ACI (10-6-2-2), ratio of transverse rebar area to center-to-center distance of transverse rebar must be higher than the minimum value defined in the standards.

$$\left(\frac{A_v}{S_v} \right)_{req} \geq \left(\frac{A_v}{S_v} \right)_{min,all}$$
$$\left(\frac{A_v}{S_v} \right)_{min,all} = max\left\{ 0.062\sqrt{f_c'}\frac{b}{f_y}, 0.35\frac{b}{f_y} \right\} \tag{15}$$

5. According to ACI (22-5-1-2), section dimensions should be selected to meet following condition:

$$V_u \leq \emptyset(V_c + 0.66\sqrt{f_c'}bd) \tag{16}$$

$$\begin{array}{lll} If\ V_s \geq 2V_c & ; & S_v \leq min\{300\ mm, d/4\} \\ Otherwise & ; & S_v \leq min\{600\ mm, d/2\} \end{array} \tag{17}$$

6. According to ACI (10-7-6-5-2), transverse rebar spacing S_v should be less than the maximum value mentioned in the standards. This ratio is:

7. According to ACI (25-7-2-2), minimum transverse rebar diameter for No.32 longitudinal rebar and less is No.10 rebar and for No.36 longitudinal rebar and more is

No.13 rebar.

8. According to ACI (25-7-2-3), stirrups should be placed such that each longitudinal rebar located in the corner and every other one with respect to corner longitudinal rebar have adequate lateral bracing with stirrup corner of maximum 135-degrees angle. Spacing of two braced longitudinal rebar should not exceed 150 mm.

As longitudinal rebar diameter affects transverse rebar spacing in two initial and terminal zones and subsequently final weight of consumed steel, the number of closed stirrups, intersecting stirrups, spacing S_0, length L_0 and steel consumption volume are calculated simultaneously for all longitudinal rebar layouts. The pattern with the lowest consumed weight is selected as the optimal response.

4. OBJECTIVE FUNCTION

This study addresses designing a square cross-section column which is a part of intermediate moment frame to reduce construction costs. Target function consists of consumed concrete, consumed steel and formwork costs. All constraints mentioned in section 3 are taken into account in the calculation of objective function. Cost function is obtained from Equation (18).

$$F(x) = b^2 L_n C_{con} + 4b L_n C_f + \gamma_s V_{st} C_{st} \tag{18}$$

where L_n is net column height, C_{con} is concrete consumption unit cost, C_f is formwork unit cost, γ_s is steel consumption specific weight, V_{st} is steel consumption volume and C_{st} is steel consumption unit cost in designed column.

5. OPTIMIZATION ALGORITHM

In this study, optimization procedure is carried out in two steps; optimizing column square cross-section and optimizing longitudinal and transverse reinforcement in a specific column section and height. So, a single-variable problem is solved in the first step using single-variable algorithms like Golden Section Search Method. This algorithm is commonly used for unconstrained problems with only one optimal point in $[a, b]$. Where a and b are lower and upper limit of response domain. Golden Section Search Method is inspired from Fibonacci search method and based on a golden ratio for search in response domain. This golden ratio is defined as $\frac{\sqrt{5}-1}{2}$ [25].

6. DESIGN EXAMPLES

6.1 Calculating column longitudinal rebar under axial load and uniaxial bending moment

In order to be able to verify the proposed method given here, and also to find exact required

longitudinal rebar in column section implementing algebraic equations, different values are assumed for axial load and bending moment according to the book appendix of [27] and the results are compared with the related design curves. Some of the studied points and dimensions in this research, mechanical properties details and results of required longitudinal rebar calculation are indicated in Table 2. Moreover, a design problem from Mc. CORMAC and BROWN's book [26] has been assessed to clarify the proposed technique.

Required longitudinal rebar cross-section is calculated for a short column in this problem. According to [26] required rebar fraction is *0.023 in* column section. The consumed rebar fraction will be *0.022* based on the calculations. This fraction for column section verifies the consistency of the suggested method response with the one in [26]. Also, *4.35 %* reduction in required steel rebar is observed in this study compared to the mentioned reference.

Table 2: Results of required longitudinal rebar calculation

Ref	Width (mm)	Mechanical Specification		Loading		Results of Present Study			
		f_c' (MPa)	f_y (MPa)	P_u (kN)	M_u (kN. m)	Target Domain	c (mm)	T (mm)	ρ
[26]*	406.4	34.47	413.68	2668.8	1084.6	4	389.86	3.319	0.022
[27]	300	35	420	2229.5	146.31	5	232.55	6.024	0.045
[27]	350	28	420	2229.5	15.60	4	401.34	2.14	0.015
[27]	450	21	400	2211.3	186.58	5	356.54	3.88	0.024
[27]	520	35	410	851.76	531.49	10	138.64	3.306	0.019
[27]	600	42	400	1360	2268	7	223.65	7.07	0.037
[27]	850	28	420	1820	2731	8	267.13	9.69	0.038
[27]	920	21	400	19195.2	735.86	3	1130.54	15.30	0.057
[27]	950	35	410	1000	9500	8	308.19	21.41	0.0778
[27]	1100	28	420	8808.8	7267.3	6	560.12	12.4	0.397

∗ concrete cover is considered *1.5 in* (*38.1 mm*).

6.2 Designing an intermediate column subjected to axial load and uniaxial bending moment

Here, a column subject to axial load and uniaxial bending moment is assumed from SP-17(14) handbook of Concrete Institute of America [17]. This column is part of an intermediate moment frame and the goal is to assess the suggested method. Details of material properties and resulting forces are shown in Table 3.

The concrete cover is 1.5 *in* and area is 24×24 *in*. As longitudinal rebar cross-section has not increased in joints, the maximum allowable rebar has been limited to $0.04A_g$. In this step, optimal design of rebar details is done based on specific column cross-section

Table 3: Second problem details

Materials and Details [17]		
Specified Yield Strength,f_y	60	ksi
Modulus of Elasticity of Steel, E_s	29000	ksi

Specified Concrete Compressive Strength , f'_c	5	ksi
Modulus of Elasticity of Concrete, E_c	4030	ksi
Net Length of the column	186	in
Length of Roof	30	in

| Loading[17] | | | |
Load Combinations	$P_u(kip)$	$M_u(kip.in)$	$V_u(kip)$
(i) U= 1.2D + 1.6L+ 0.5S	890	0	0
(ii) U= 1.2D +1.0W + 1.0L+0.5S	800	651	5
(iii) U= 1.2D+1.0E+1.0L+0.2S	818	2401	18
(iv) U= 0.9D+1.0E+1.0L+0.2S	486	2401	18

As column is part of an intermediate moment frame, the constraints mentioned in section 3.2. are applied to calculate the maximum shear in column.
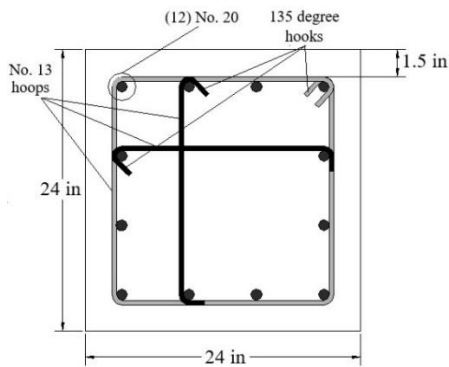
Figure 3a. Section details of present research
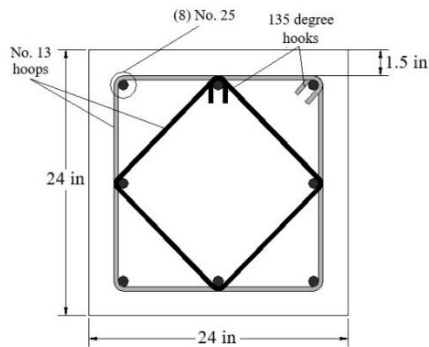
Figure 3b. Section details of reference [17]
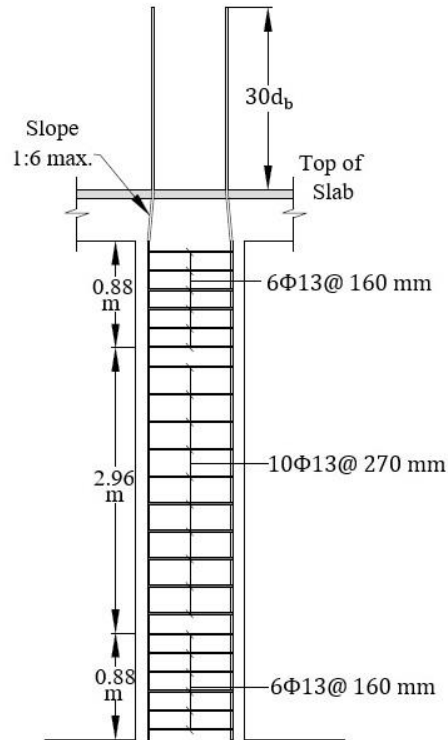
Figure 3c. Details of transverse based on section width from [17]

Figure 3. Design results based on section width from [17]

Load combinations used for analysis in Ref. [17] are shown in Table 3. Load combinations include earthquake are, combination (iii) and (iv). For maximum axial load and bending moment from load combinations of Table 3 and based on defined equilibrium equations, neutral axis is located in domain 5 in Table 1 and its height is $19.67\ in$. Steel plate thickness is obtained $-0.108\ in$ after solving equilibrium equations in domain 5

considering mentioned neutral axis height. Regarding negative thickness calculated, it is observed that the column with mentioned cross-section does not require additional longitudinal rebar and concrete is able to carry total load. So, minimum longitudinal rebar is $0.01A_g$ based on the standards.

There are 64 possible states for longitudinal rebar placement in column section based on longitudinal rebar diameter, minimum allowable spacing between longitudinal rebar and concrete cover. Longitudinal and transverse rebar results in this study and the results from [17] are shown in Fig. 3a-3b. Transverse rebar details in section one are shown in Fig. 3c. Transverse rebar spacing is rounded off to multiplications of *5mm* because of constructional structure. Steel specific weight is 78.5 (kN/m^3). Consumes steel weight is calculated $1.499\ kN$ considering dowel rebar length and roof height, while consumed steel weight is $1.984\ kN$ in [17]. The proposed method indicates *24%* reduction in consumed steel weight. Points of axial load and bending moment are shown in interaction curve in Fig. 4. As transverse rebar diameter directly affects spacing and longitudinal rebar diameter, the proposed method can obtain the minimum consumed steel weight under the stated conditions.

In the next step column section cost is optimized by Golden Section Search Method [25]. Column section dimension is considered as a design variable in mentioned algorithm and is selected based on applied loads. Concrete, steel and formwork unit cost are respectively $100(USD/m^3)$, $87(USD/kN)$ and $5(USD/m^2)$.

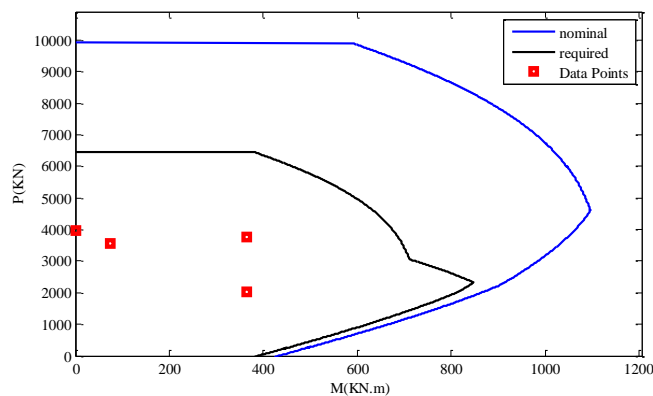
Figure 4. Points of applied loads for the section in [17] in interaction curve

Column section dimension is *300-1500 mm* with *5 mm* step. Optimal design details of dimension and reinforcement are shown in Fig. 5. Points of applied loads optimized with section dimensions different from [17] and implementation of Golden Section Search method are shown in interaction curve in Fig. 6.
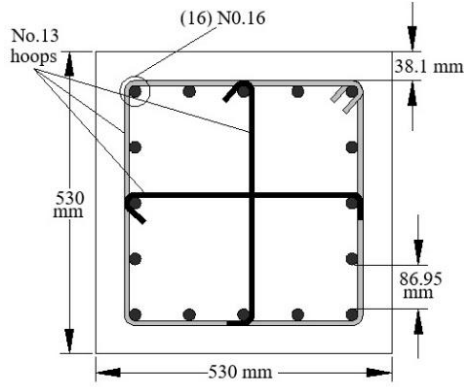
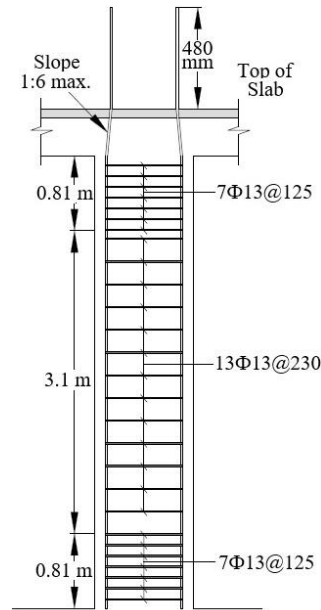Figure 5a. Optimal response section details

Figure 5b. Optimal response transverse rebar spacing details

Figure 5. Optimal design details for Golden Section Search Method Algorithm

Fig. 6 indicates the most optimal dimensions and values for longitudinal rebar. Consumed steel in optimized design is equal to $1.3359\ kN$. Design weight ratio based on SP-17(14) shows *32.66%* reduction in consumed steel. Steel, concrete and formwork costs are shown in Table 4. According to Table 4, this approach in intermediate column optimal design showed *26%* reduction in reinforced concrete construction cost.
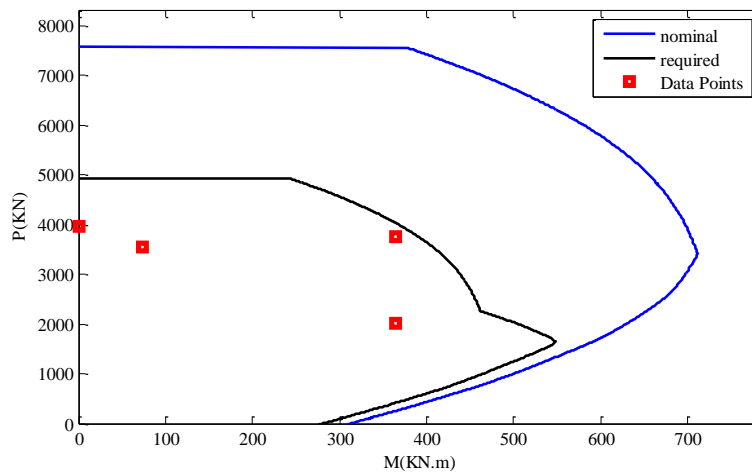
Figure 6. Points of applied loads to the optimized column section by using Golden Section Search Method Algorithm in interaction curve

Table 4: Steel, concrete and formwork costs

Method	Width (mm)	Quantity			Cost of components			Optimum Cost (USD)
		Concrete (m^3)	Formwork (m^2)	Steel (kN)	Concrete (USD)	Formwork (USD)	Steel (USD)	
Present Study	530	1.31	10.01	1.3359	131	50.05	116.22	297.27
Ref. [17]	609.6	1.724	11.52	1.984	172.4	57.6	172.61	402.61

3.6 Effect of reinforced column cost optimization on dimensions and consumed steel

To investigate the effect of optimization on intermediate concrete column cost considering axial load and bending moment, a column subjected to variable axial load and bending moment is studied in this section. Typically, axial load and bending variation domains in common columns in RC frames are respectively *1000-11000 kN* and *200-1000 kN.m* [5]. Also, shear force is calculated according to ACI (18-4-3-1). Material details, loading and material costs are presented in Table 5. Maximum and minimum consumed longitudinal rebar in column section are limited to *0.01* and *0.04* respectively. According to the standards, concrete minimum cover is *40 mm*. longitudinal and transverse rebar dimensions are the same as previous problem.

Table 5: Details of third problem

Materials and Details:			
Specified Yield Strength,f_y		400	MPa
Modulus of Elasticity of Steel, E_s		200000	MPa
Specified Concrete Compressive Strength , f_c'		35	MPa
Net Length of Column		3.5	m
Length of Roof		0.4	m
Column Width (mm)	Lower bound	Upper bound	Step
	300	1500	5
Loading:			
	Lower bound	Upper bound	Step
$P_u(kN)$	1000	11000	2000
$M_u(kN.m)$	200	1000	200
Cost Parameters:			
Cost of Concrete (USD/m^3)	Cost of Steel (USD/kN)	Cost of Formwork (USD/m^2)	
73	100	7	

Fig. 7a shows the variation of reinforced concrete column with respect to optimization based on reinforced concrete column construction cost considering variation of axial load and bending moment in defined limit. As it is clear in Fig. 7a, promotion of axial load or bending moment or both results in an increase in optimal reinforced concrete column section.
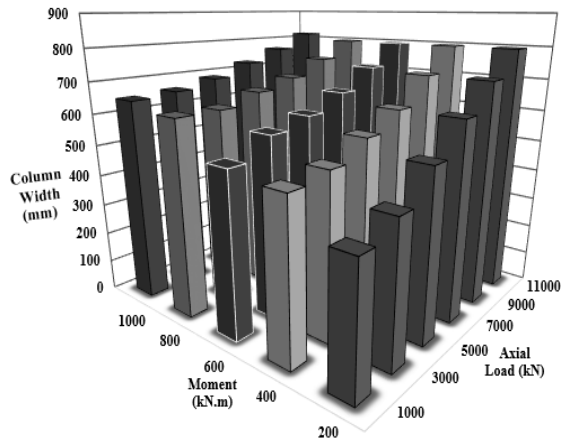
$$Width(P_u, M_u) = -2.976 \times 10^{-5}M_u^2 - 3.06$$
$$\times 10^{-5}P_uM_u + 1.214 \times 10^{-6}P_u^2$$
$$+ 0.35M_u + 0.03027P_u + 314.3;$$
$$R^2 = 0.989 \quad, \quad RMSE = 12.53$$

Figure 6a. Column dimension based on the variation of axial load and bending moment
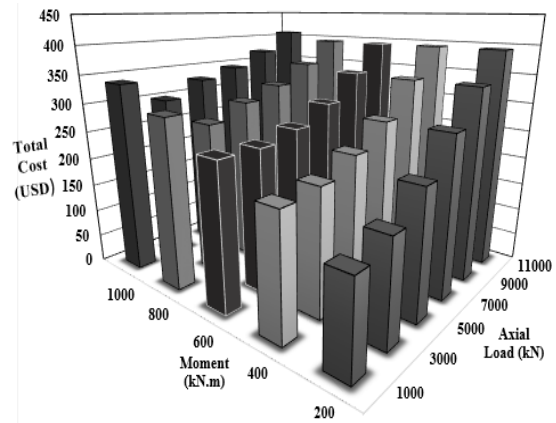
$$COST(P_u, M_u) = 1.135\,M_u^3 + 4.08P_uM_u^2 + 2.651P_u^2M_u$$
$$-7.414P_u^3 + 2.028M_u^2 - 20.52P_uM_u$$
$$+18.07P_u^2 + 25.74M_u + 61.2P_u + 284;$$
$$R^2 = 0.995 \quad, \quad RMSE = 5.679$$

Figure 6b. optimal response total cost variation based on the variation of axial load and bending moment

As shown in bar chart in Fig. 6b, variation of construction cost in a variable axial load domain of *1000-11000 kN* and constant bending moment of *200 kN.m* or *4000 kN.m* is absolutely ascendant. While for bending moments of *600 kN.m*, *800 kN.m* and *1000 kN.m*, construction cost of reinforced concrete column is a curve with initially descendant behavior and ascendant behavior in following. The reason is increase in eccentricity and interring tension-controlled zone of interaction curve. In this state, the section is under the effect of larger bending moment comparing to axial load and neutral axis is located in domains 9, 10 or 11 of Table 1. Also the section requires more longitudinal rebar to meet equilibrium equations considering effective strength reduction factor in solution of equations in tension-controlled, transition and compression-controlled zones. This rises the final construction cost of reinforced concrete column. Decrease in eccentricity and getting away from tension-controlled zone reduced the required longitudinal rebar. Subsequently, axial load goes up, section inters pressure-controlled zone and stays in this state. The cost variation is ascendant since axial load increases. Equations fitting bar charts of cost variation and optimized reinforced concrete column are demonstrated in Figs. 6a and 6b. Correlation coefficient results indicate suitable consistency of resulting equations and data obtained from charts. It should be mentioned that presented equations and charts for a presumed column in present case are easily applicable for different heights of reinforced concrete column.

Fig. 7a and 7b present variation of longitudinal rebar section and optimal column dimension with respect to variation of axial load in constant bending moment of *1000 kN.m* and variation of bending moment in constant axial load of *1000 kN*. According to Figs. 7a and 7b, to obtain reinforced concrete column with the lowest construction cost, growth in reinforced concrete column dimension prevents from increase in longitudinal rebar section. Therefore, longitudinal rebar section always stays close to the minimum allowable

longitudinal rebar. This is because of high cost difference between consumed steel and concrete. So, in order to design a reinforced concrete column with the lowest construction cost, it is recommended that the dimensions be chosen to keep required steel rebar close to the minimum allowable value.

In addition to the effect of axial load and bending moment on optimal design of reinforced concrete column, the effect of concrete compression strength variation on optimal design of intermediate reinforced concrete column is investigated under constant bending moment *1000 kN.m* and variable axial load of *1000 kN* to *11000 kN*. The utilized compressive strengths and their construction costs are presented in Table 6.

Table 6: Consumed concrete cost for different compressive strengths

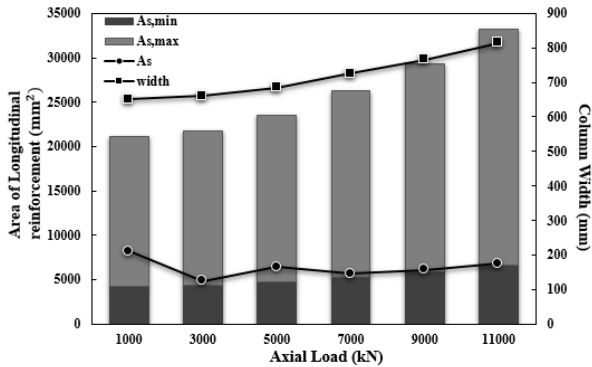| Specified Strength, f'_c (MPa)	Cost of Concrete (USD/m^3)
20	41
30	63
35	73
40	84
50	105

Figure 7a. Variation of longitudinal rebar section and optimal column dimension with respect to variation of axial load in constant bending moment of *1000 kN.m*
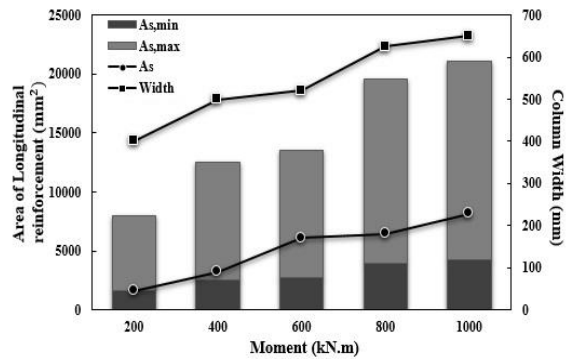
Figure 7b. Variation of longitudinal rebar section and optimal column dimension with respect to variation of bending moment in constant axial load of *1000 kN*

Fig. 8a-d respectively show dimension variation of optimal column, optimal consumed longitudinal rebar, optimal construction cost and total consumed steel cost in optimal column under variable axial load, variable characteristic compressive strength of concrete and constant bending moment of *1000 kN.m*.

Variation of optimal column dimension for concrete characteristic compressive strength of *20 MPa* has sharper slope comparing to other studied strengths in Fig. 8a Concrete with characteristic compressive strength of *20 MPa* has the lowest consumed steel weight, construction cost and total consumed steel in axial load of *1000 kN*, although its section dimensions are larger than others. With an increase in axial load in strengths of *30 MPa* to *50 MPa*, longitudinal rebar fraction diminishes and get closer to the minimum value. Also,

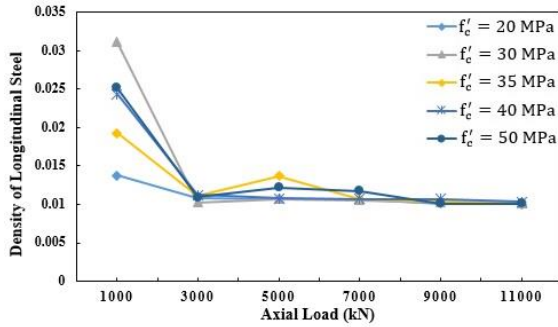section dimension increases with a mild slope.

Figure 8. Variation of optimal longitudinal rebar fraction under variable axial load, variable characteristic compressive strength of concrete and bending moment of *1000 kN.m*
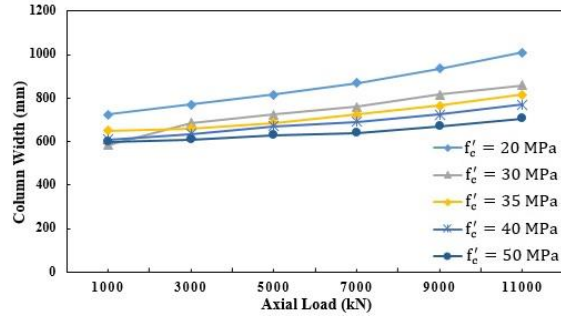

Figure 8a. Variation of optimal column dimension under variable axial load, variable characteristic compressive strength of concrete and bending moment of *1000 kN.m*
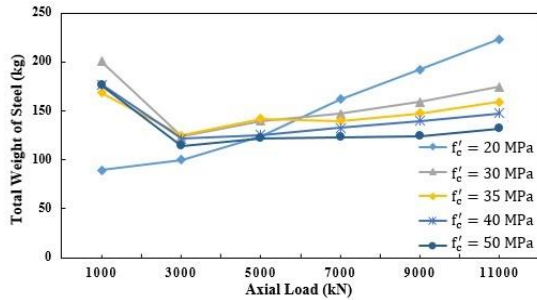

Figure 8d. Variation of total consumed steel cost in optimal column under variable axial load, variable characteristic compressive strength of concrete and bending moment of *1000 kN.m*
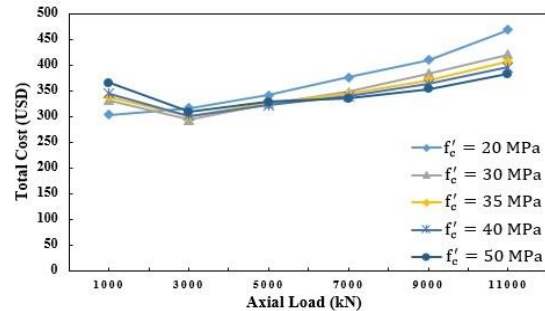

Figure 8c. Variation of optimal construction cost under variable axial load, variable characteristic compressive strength of concrete and bending moment of *1000 kN.m*

So, as Figs. 8c and 8d show, total consumed steel weight and final construction cost decline. This reduction in cost and consumed steel weight continues with an increase in axial load to *3000 kN*. In axial load of *3000 kN* to *11000 kN*, concrete with compressive strength of *20 MPa* has the highest steel consumption and construction cost. Also as concrete compressive strength goes up, two mentioned parameters intensely decrease. For an intermediate column subjected to low axial load of around *3000 kN*, using concrete with compressive strength of *20 MPa* can demote the costs. For a column subjected to an axial load of more than *3000 kN*, using concrete with higher strength reduces total steel weight as well as construction cost of intermediate reinforced concrete column.

7. CONCLUSION

Implementing equivalent equations, an exact solution was derived in this study for the intensity of longitudinal steel requirement of square section columns. Longitudinal rebar cross-

sections were discretized based on catalogue regarding standard longitudinal and transverse rebar dimensions. The cross sections were then designed considering all intermediate deformability constraints according to ACI318-14 Code of Practice and optimal response was obtained from the illustrated interaction curve. The method applied in this study brought about a *4.35 %* reduction in longitudinal rebar cross-section, *32.66%* reduction in total steel consumption and *26%* reduction in construction cost of reinforced concrete column. It may be incorporated as a suitable approach in designing columns of intermediate frames. The effects of variation in axial load, bending moment and concrete compressive strength on dimension characteristics of optimal column, construction cost and total consumed steel were investigated then. According to the results, designing optimized dimension to stay close to the minimum allowable value in the standards can suitably give a column with the lowest construction cost. Also concrete with compressive strength of *20 MPa* under an inconstant axial load of *1000-3000 kN* can be optimized column for the lowest cost possible. As well, a concrete cross section with a compressive strength of *30 MPa* and higher under variable axial load of *3000-11000 kN* can be a suitable choice for an optimum column carrying the lowest consumed steel weight and construction cost as possible.

REFERENCES

1. Aschheim M, Hernández-Montes E, Gil-Martín LM. Design of optimally reinforced RC beam, column, and wall sections, *J Struct Eng 2008*; **134**(2): 231-9.
2. Aschheim M, Hernández-Montes E, Gil-Martín LM. Optimal domains for strength design of rectangular sections for axial load and moment according to Eurocode 2, *Eng Struct* 2007; **29**(8): 1752-60.
3. Whitney CS, Cohen E. Guide for ultimate strength design of reinforced concrete, *ACI J* 1956; **53**(11): 455-90.
4. Hernández-Montes E, Aschheim M, Gil-Martin LM. Impact of optimal longitudinal reinforcement on the curvature ductility capacity of reinforced concrete column sections, *Mag Concrete Res* 2004; **56**(9): 499-512.
5. Park HS, Lee H, Kim Y, Hong T, Choi SW. Evaluation of the influence of design factors on the CO2 emissions and costs of reinforced concrete columns, *Energ Build* 2014; **82:** 378-84.
6. MacGregor JG, Wight JK. *Reinforced Concrete: Mechanics and Design*, Prentice Hall, New Jersey, USA, 2009.
7. Lee HJ, Aschheim M, Hernández-Montes E, Gil-Martín LM. Optimum RC column reinforcement considering multiple load combinations, *Struct Multidiscip O* 2009; **39**(2): 153-70.
8. Esfandiary MJ, Sheikholarefin S, Bondarabadi HR. A combination of particle swarm optimization and multi-criterion decision-making for optimum design of reinforced concrete frames, *Int J Optim Civil Eng* 2016; **6**(2): 245-68.
9. Kaveh A, Farhoudi N. Layout optimization of braced frames using differential evolution algorithm and dolphin echolocation optimization, *Period Polytech Civil Eng* 2015; **59**(3): 441-9.

10. Mahallati-Rayeni A, Ghohani-Arab H, Ghasemi MR. Optimization of steel moment frame by a proposed evolutionary algorithm, *Int J Optim Civil Eng* 2018; **8**(4): 511-24.
11. Kaveh A, Ghazaan MI, Bakhshpoori T. An improved ray optimization algorithm for design of truss structures, *Period Polytech Civil Eng* 2013; **57**(2): 97-112.
12. Ghasemi MR, Farshchin M. Ant colony optimisation-based multiobjective frame design under seismic conditions, *Proceedings of the Institution of Civil Engineers - Structures and Buildings* 2011; **164**(6): pp. 421-432.
13. Kaveh A, Zakian P. Optimal seismic design of reinforced concrete shear wall-frame structures, *KSCE Civil Eng* 2014; **18**(7): 2181-90.
14. Kaveh A, Vaez SH, Hosseini P. Modified dolphin monitoring operator for weight optimization of frame structures, *Period Polytech Civil Eng* 2017; **61**(4): 770-9.
15. Kaveh A, Sabzi O. A comparative study of two meta-heuristic algorithms for optimum design of reinforced concrete frames, *Int J Civil Eng* 2011; **9**(3): 193-206.
16. Kaveh A, Sabzi O. Optimal design of reinforced concrete frames using big bang-big crunch algorithm, *Int J Civil Eng* 2012; **10**(3): 189-200.
17. Kaveh A, Zakian P. Seismic design optimisation of RC moment frames and dual shear wall-frame structures via CSS algorithm, *Asian J Civil Eng (BHRC)* 2014; **15**(3): 435-65.
18. Adamu A, Karihaloo BL. Minimum cost design of RC frames using the DCOC method Part I: columns under uniaxial bending actions, *Struct Optim* 1995; **10**(1): 16-32.
19. de Medeiros GF, Kripka M. Optimization of reinforced concrete columns according to different environmental impact assessment parameters, *Eng Struct* 2014; **59**: 185-94.
20. Sánchez-Olivares G, Tomás A. Improvements in meta-heuristic algorithms for minimum cost design of reinforced concrete rectangular sections under compression and biaxial bending, *Eng Struct* 2017; **130**: 162-79.
21. Gharehbaghi S, Fadaee MJ. Design optimization of RC frames under earthquake loads, *Int J Optim Civil Eng* 2012; **2**(4): 459-77.
22. Gholizadeh S, Aligholizadeh V. Optimum design of reinforced concrete frames using bat meta-heuristic algorithm, *Int J Optim Civil Eng* 2013; **3**(3): 483-97.
23. Gheyratmand C, Gholizadeh S, Vababzadeh B. Optimization of RC frames by an improved artificial bee colony algorithm, *Int J Optim Civil Eng* 2015; **5**(2): 189-203.
24. Aschheim M, Hernández-Montes E, Gil-Martín LM. Design of optimally reinforced RC beam, column, and wall sections, *J Struct Eng* 2008; **134**(2): 231-9.
25. Snyman JA. *Practical Mathematical Optimization*, Springer, USA, 2005.
26. McCormac JC, Brown RH. *Design of Reinforced Concrete*, John Wiley & Sons, 9th edition, USA, 2015.
27. Mostofinejad D. *Reinforced Concrete Structures*, 2nd edition, Esfahan, Iran, 2015.