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EFFICIENCY EVALUATION OF PROPOSED OBJECTIVE FUNCTIONS IN STRUCTURAL DAMAGE DETECTION BASED ON MODAL STRAIN ENERGY AND FLEXIBILITY APPROACHES

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ABSTRACT

Civil infrastructures such as bridges and buildings are prone to damage as a result of natural disasters. To understand damages induced by these events, the structure needs to be monitored. The field of engineering focusing on the process of evaluating the location and the intensity of the damage to the structure is called Structural Health Monitoring (SHM). Early damage prognosis in structures is the fundamental part of SHM. In fact, the main purpose of SHM is obtaining information about the existence, location, and the extent of damage in the structure. Since numerous structural damage detection problems can be solved as an inverse problem based on the proposed objective functions by using optimization algorithm, in this paper, related studies are investigated which discussing objective functions based on Modal Strain Energy (MSE) and flexibility methods including Modal Flexibility (MF), and Generalized Flexibility Matrix (GFM). To illustrate the extent of effectiveness of these objective functions based on the above-mentioned modal parameters, an efficiency index called Impact Factor (IF) is defined. Finally, the best objective function is introduced for each numerical case study based on IF by means of evaluating the obtained result.

Keywords: structural health monitoring (SHM); objective function; modal strain energy (MSE); modal flexibility (MF); generalized flexibility matrix (GFM); impact factor (IF).

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1. INTRODUCTION

Due to the deterioration of the structures over time, their service life constructed by mankind is limited. Some have already reached the end of their predicted life cycle. This alters the structure's characteristics and performance. The basic concept of damage detection can be defined as a method in which the variations of structure's characteristic under damage can be determined. Deterioration in general and other factors such as earthquakes and impact loads have various effects on a structure's performance level. Disregarding these factors can result in structural damage, considerable financial losses, and more importantly heavy casualties. Early damage detection and prevention of further progress not only prevents catastrophic failures of structures but also adds to their useful life. For this reason, many researchers have attempted to introduce new structural damage detection methods in the past two decades. The basic assumption for all methods is that damage results in stiffness alterations in the damaged element. These damages are mainly the result of structure excitation and recorded responses under these reactions [1].

Generally, structural damage detection methods are divided into static damage identification methods and dynamic ones. Static identification methods are rarely used. This is mainly due to the demanding extensive amount of measured data. Furthermore, these methods require an updated Finite Element (FE) model with accurate material characteristics and static load tests which cause perturbation in the operation of the structure. In contrast, dynamic-based identification methods including vibration-based damage identification methods are more applicable and efficient than the static ones [2]. In order to identify structural damage, various vibration-based damage detection techniques use natural frequencies and mode shape changes based on the results of the obtained modal structures analysis [3]. Although both changes, the natural frequencies and the mode shapes, can be used for structural damage detection, there are some downsides to these methods. To begin with, identifying damage occurrence utilizing natural frequency alterations with high accuracy is feasible, whereas localizing and quantifying the damage is usually impractical. Furthermore, using changes of mode shapes requires more measurements in various points of the structure to conveniently identify the occurrence and the location of the damage. Therefore, researchers turned to novel and more efficient methods based on modal data such as Modal Strain Energy (MSE) and Modal Flexibility (MF).

On the other hand, vibration-based damage identification can be considered as an inverse problem which has turned into a popular topic for researchers in recent years [4-15]. This process consists of two methods namely hard computing and soft computing methods. Relative to soft computing methods, hard computing methods are not suitable due to the employment of complicated mathematical concepts, the complexity of the problem, high sensitivity of entry data, and lack of accurate convergence to the desired answer [16]. In this regard, lately, many researchers have been using soft computing approaches based on finite element model updating to avoid confronting the above-mentioned challenges. For this purpose, objective functions with sufficient sensitivity to damage were introduced using optimization algorithms with high accuracy and convergence speed. For instance, researchers have used Genetic Algorithm (GA) [17], Particle Swarm Optimization (PSO) [18, 19], and many other optimization algorithms to solve damage detection problems as inverse ones [20-24].

In this paper, first of all, an efficiency indicator called Impact Factor (IF) is proposed to evaluate and compare objective functions of studies based on MSE, MF, and GFM alongside their numerical examples. These numerical studies include beam-like structure, shear frame, moment-resisting frame, planar truss, and spatial truss. Finally, the results from numerical studies are illustrated by applying IF to the objective function of each study, and the best objective function of each study is specified separately for each type of structure.

2. THE PROPOSED INDEX

Since numerous damage detection problems deal with damage localization and quantification using an objective function, in this section, an efficiency index named IF is presented to indicate the competency of the proposed objective functions based on numerical studies. Consequently, the following assumptions are taken into consideration:

The effective parameters influencing the objective function of a paper in each of its corresponding case studies are as follows: number of elements in the finite element model, minimum number of vibrating modes of the each test example which are used to identify damage, the maximum value of noise level which can be applied to natural frequencies, mode shapes, or both of them simultaneously, and maximum number of damaged elements at one of its damage scenario.

Each effective parameter is normalized with a maximum and minimum value of the corresponding effective parameter among related numerical studies. These parameters have equal effects.

The objective function of each paper in any of its case studies among all related test examples will receive the highest IF if it has simultaneously maximum number of elements in the finite element model, maximum number of damaged elements in one of its damage scenarios, maximum value of noise level, and also minimum number of vibrating modes for identifying damage. This indicates that the employed objective function of the corresponding research is efficient and robust.

The applied IF to all objective functions are specifically defined for each structure type and compared with each other later. In other words, the number of investigated case studies are equal to the number of objective functions in different structure types.

The value of noise level which can be applied to the natural frequencies and mode shapes of a structure is presented as follows:

$$\xi = \xi_f + \xi_\phi \tag{1}$$

where ξ_f and ξ_Φ are the applied noise to the natural frequencies and mode shapes of the structure in percentage, respectively.

Finally, taking the above-mentioned assumptions into account, the applied IF to check the efficiency and the sturdiness of the employed objective function of a paper in each of its numerical studies is defined as:

$$IF = \frac{N}{N_{\max}} + \frac{M_{\min}}{M} + \frac{\xi}{\xi_{\max}} + \frac{D}{D_{\max}} \tag{2}$$

where N is the considered number of the elements for the finite element model, M is the minimum number of the considered vibrating modes to detect damage, ξ is the maximum value of noise level in percentage, and D is the maximum number of damaged elements in one of the damage scenarios of test example. Moreover, it should be pointed out that all 4 terms in Eq. (2) have equal effects, and maximum value of each term is 1. Consequently, by substituting Eq. (1) in Eq. (2) and applying 0.5 to both parts of noise level term, Eq. (2) can be written as follows:

$$IF = \frac{N}{N_{\max}} + \frac{M_{\min}}{M} + \frac{1}{2} \times \left(\frac{\xi_f}{\xi_{f\max}} + \frac{\xi_\Phi}{\xi_{\Phi\max}} \right) + \frac{D}{D_{\max}} \tag{3}$$

where the subscript min and max are denoting the corresponding minimum and maximum values in each type of the numerical studies, respectively. Furthermore, in Eq. (3), each term is normally distributed between 0 and 1.

The mean value of IF for each modal parameter (MSE, MF, and GFM) in each type of case study, can be calculated as follows:

$$\overline{IF}_m = \frac{\sum_{i=1}^{n} IF_i}{n} \tag{4}$$

where n is the number of case studies based on the corresponding modal parameter. In Eq. (4), m refers to the type of the modal parameter, thus $\overline{IF}_m$ may be mentioned as $\overline{IF}_{MF}$,$\overline{IF}_{GFM}$ and $\overline{IF}_{MSE}$.

In the following sections, the proposed indicator is calculated separately for different numerical studies of investigated papers with objective functions based on three modal parameters (MSE, MF, and GFM).

3. NUMERICAL STUDIES

In this section, 61case studies from 30 papers are presented, as shown in Table 1. These cases are categorized into 5 group namely beam-like structures, shear frame, moment-resisting frame, planar truss, and spatial truss. The objective functions of these papers are based on MSE, MF, and GFM which are classified into the abovementioned groups of case studies. Fig. 1 illustrates these classifications. In each of these 5 categories, the frequency of MSE, MF and GFM are compared based on their appearance in the reviewed studies. Finally, by using Eq. (3), the proposed IF indicator and subsequently the best objective function for each group is given separately.

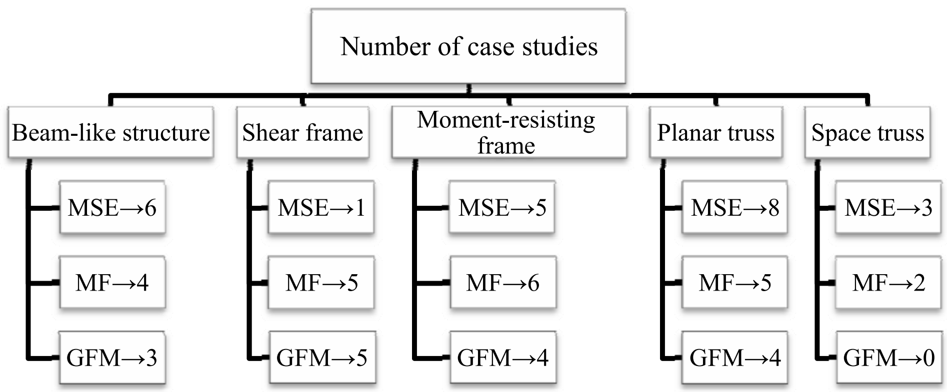

Figure 1. Number of case studies based on three modal parameters at five different types of the examined structures

Table 1: Characteristics of the investigated articles

Article number	Article name	Authors	Published year	Journal
1	Benchmark Studies for Bridge Health Monitoring Using an Improved Modal Strain Energy Method [4]	Parviz Moradipour, Tommy H.T. Chan, Chaminda Gallage	2017	Procedia Engineering
2	Damage Detection Method Based on Element Modal Strain Energy Sensitivity [5]	Wang-Ji Yan, Tian-Li Huang, Wei-Xin Ren	2010	Advances in Structural Engineering
3	Damage detection of shear buildings using deflections obtained by modal flexibility [6]	K Y Koo, S H Sung, J W Park, H J Jung	2010	smart materials and structures
4	Damage prognosis by means of modal residual force and static deflection obtained by modal flexibility based on the diagonalization method [7]	Gholamreza Ghodrati Amiri, Ali Zare Hosseinzadeh, Abdollah Bagheri, and Ki-Young Koo	2013	smart materials and structures
5	An efficient indicator for structural damage localization using the change of strain energy based on static noisy data [8]	S. M. Seyedpoor, O. Yazdanpanah	2014	Applied Mathematical Modelling
6	Enhanced optimization-based structural damage detection method using modal strain energy and modal frequencies [9]	M. R. Ghasemi, M. Nobahari, N. Shabakhty	2018	Engineering with Computers

Article number	Article name	Authors	Published year	Journal
7	A generalized flexibility matrix-based model updating method for damage detection of plane truss and frame structures [10]	Leila Katebi, Mohsen Tehranizadeh, Negar Mohammadgholibeyki	2018	Journal of Civil Structural Health Monitoring
8	structural damage detection based on generalized flexibility matrix [11]	Jing Li	2012	Advanced Materials Research
9	Structural damage detection from modal strain energy change [12]	ZY Shi, SS Law, LM Zhang	2000	journal of engineering mechanics
10	Structural damage detection using sparse sensors installation by optimization procedure based on the modal flexibility matrix [13]	A. Zare Hosseinzadeh, G. Ghodrati Amiri, S.A. Seyed Razzaghi, K.Y. Koo, S.H. Sung	2016	Journal of Sound and Vibration
11	structural damage localization from modal strain energy change [14]	Z.Y. Shi, S.S. Law, L.M. Zhang	1998	Journal of Sound and Vibration
12	Structural multi-damage identification based on modal strain energy equivalence index method [15]	HY Guo, ZL Li	2014	International Journal of Structural Stability and Dynamics
13	Damage assessment via modal data with a mixed particle swarm strategy [18]	A. Kaveh, S.M. Javadi, and M. Maniat	2014	Asian journal of civil engineering
14	Two-stage damage identification based on modal strain energy and revised particle swarm optimization [19]	SL Ma, SF Jiang, LQ Weng	2014	International Journal of Structural Stability and Dynamics
15	Application of Generalized Flexibility Matrix in Damage Identification using Imperialist Competitive Algorithm [20]	Mehdi Masoumi, Ehsan Jamshidi, Mahdi Bamdad	2015	KSCE Journal of Civil Engineering
16	Cyclical Parthenogenesis Algorithm for guided modal strain energy based structural damage detection [21]	A. Kaveh, A. Zolghadr	2017	Applied Soft Computing
17	Efficiency of Jaya algorithm for solving the optimization-based structural damage identification problem based on a hybrid objective function [22]	Dinh-Cong Du, Ho-Huu Vinh, Vo-Duy Trung, Ngo-Thi Hong Quyen & Nguyen-Thoi Trung	2018	Engineering Optimization

Article number	Article name	Authors	Published year	Journal
18	A damage identification method for truss structures using a flexibility-based damage probability index and differential evolution algorithm [25]	SM Seyedpoor, M Montazer	2016	Inverse Problems in Science and Engineering
19	A flexibility method for structural damage identification using continuous ant colony optimization [26]	Maryam Daei, S. Hamid Mirmohammadi	2015	Multidiscipline Modeling in Materials and Structures
20	A flexibility-based method via the iterated improved reduction system and the cuckoo optimization algorithm for damage quantification with limited sensors [27]	Ali Zare Hosseinzadeh, Abdollah Bagheri, Gholamreza Ghodrati Amiri, and Ki-Young Koo	2014	Smart Materials and Structures
21	A generalized flexibility matrix based approach for structural damage detection [28]	Jing Li, Baisheng Wu, Q.C. Zeng, C.W. Lim	2010	Journal of Sound and Vibration
22	A new two-stage method for damage identification in linear-shaped structures via Grey System Theory and optimization algorithm [29]	Gholamreza Ghodrati Amiri, Ali Zare Hosseinzadeh, Mojtaba Jafarian Abyaneh	2015	Journal of Rehabilitation in Civil Engineering
23	An improved hybrid optimization algorithm for vibration based-damage detection [30]	Idilson António Nhamage, Rafael Holdorf Lopez, Leandro Fleck Fadel Miguel	2016	Advances in Engineering Software
24	Closed-form modal flexibility sensitivity and its application to structural damage detection without modal truncation error [31]	Wang-Ji Yan, Wei-Xin Ren	2014	Journal of Vibration and Control
25	Generalized flexibility-based model updating approach via democratic particle swarm optimization algorithm for structural damage prognosis [32]	G. Ghodrati Amiri, A. Zare Hosseinzadeh, S. A. Seyed Razzaghi	2015	international journal of optimization in civil engineering
26	Model-based identification of damage from sparse sensor measurements using Neumann series expansion [33]	Ali Zare Hosseinzadeh, Gholamreza Ghodrati Amiri, Seyed Ali Seyed Razzaghi	2017	Inverse Problems in Science and Engineering

Article number	Article name	Authors	Published year	Journal
27	Multi-stage approach for structural damage identification using modal strain energy and evolutionary optimization techniques [34]	V. Srinivas, K. Ramanjaneyulu, C. Antony Jeyasehar	2010	Structural Health Monitoring
28	Optimization-based method for structural damage localization and quantification by means of static displacements computed by flexibility matrix [35]	Ali Zare Hosseinzadeh, Gholamreza Ghodrati Amiri, Ki-Young Koo	2016	Engineering Optimization
29	Structural Damage Detection Using Generalized Flexibility Matrix and Changes in Natural Frequencies [36]	Jing Li, Zhengguang Li, Huixiang Zhong and Baisheng Wu	2012	AIAA journal
30	Structural Damage Detection Using Parameters Combined with Changes in Flexibility Based on BP Neural Networks [37]	Zhang Jun	2011	Advanced Materials Research

3.1 Beam-like structures

13 case studies (MSE=6, MF=4, GFM=3) in 12 related papers [5, 11, 14, 19, 21, 26-28, 31, 34, 36, 37] are investigated by applying IF to their objective functions. In these structures, the number of papers with objective functions based on MSE (46.2%) are more than those based on MF (30.8%) and GFM (23%) as can be seen in Fig. 2. The maximum and minimum values of effective parameters among all beam-like structures (13 case studies) are shown in Table 2 By substituting these maximum and minimum values of effective parameters in Eq. (3) with values from Table 2, IF_{beam} is obtained as follows:

$$IF_{beam} = \frac{N}{40} + \frac{1}{M} + \frac{1}{2} \times \left(\frac{\xi_f}{30} + \frac{\xi_\Phi}{30} \right) + \frac{D}{15} \tag{5}$$

Table 2: The maximum and minimum values of effective parameters in beam-like structure

N_{max}	M_{min}	ξ_{fmax} (%)	$\xi_{\Phi max}$ (%)	D_{max}
40	1	30	30	15

As depicted in Table 3 through Table 5, by putting N, M, ξ_f, ξ_Φ, and D in Eq. (5), the best objective functions among all beam-like structures based on MSE, MF, and GFM are obtained. The minimum, mean, and maximum values of IF for each modal parameter in beam-like structures are shown in Table 6.

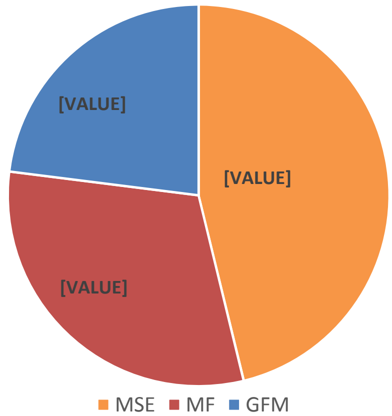

Figure 2. Frequency of examined modal parameters of the beam-like structures

Table 3: Obtained values of IF_{beam} in objective functions based on MSE

Article number	N	M	ξ_f (%)	ξ_Φ (%)	D	$\frac{N}{40}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{30}+\frac{\xi_\Phi}{30}\right)$	$\frac{D}{15}$	IF_{beam}
16	20	5	0	0	5	0.50	0.20	0.00	0.33	1.03
2	15	1	0	1	5	0.38	1.00	0.02	0.33	1.73
2	30	1	0	5	6	0.75	1.00	0.08	0.40	IF_{max}=2.23
27	11	5	2	10	3	0.28	0.20	0.20	0.20	0.88
11	8	5	0	0	1	0.20	0.20	0.00	0.07	IF_{min}=0.47
14	40	4	0	0	1	1	0.25	0.00	0.07	1.32
$\overline{IF}_{MSE}=1.28$										

Table 4: Obtained values of IF_{beam} in objective functions based on MF

Article number	N	M	ξ_f (%)	ξ_Φ (%)	D	$\frac{N}{40}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{30}+\frac{\xi_\Phi}{30}\right)$	$\frac{D}{15}$	IF_{beam}
19	3	5	0	0	1	0.75	0.20	0.00	1.00	1.95
20	2	1	5	0	3	0.50	1.00	0.08	0.20	1.78
24	2	5	30	30	6	0.50	0.20	1.00	0.40	IF_{max} =2.10
30	1	1	0	0	1	0.25	0.10	0.00	0.07	IF_{min} =0.42
$\overline{IF}_{MF}=1.56$										

Table 5: Obtained values of IF_{beam} in objective functions based on GFM

Article number	N	M	ξ_f (%)	ξ_Φ (%)	D	$\frac{N}{40}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{30}+\frac{\xi_\Phi}{30}\right)$	$\frac{D}{15}$	IF_{beam}
21	20	1	1	5	3	0.50	1.00	0.10	0.20	IF_{max} =1.80
29	20	1	1	5	2	0.50	1.00	0.10	0.13	1.73
8	20	1	0	0	2	0.50	1.00	0.00	0.13	IF_{min} =1.63
$\overline{IF}_{GFM}=1.72$										

Table 6: The minimum, mean, maximum values of IF_{beam} for each modal parameter

	Modal parameters	IF_{min}	IF_{max}	$\overline{IF}_m$
Beam-like structures	MSE	0.47	2.23	1.28
	MF	0.42	2.10	1.56
	GFM	1.63	1.80	1.72

3.2 Shear frames

From 8 related papers [6, 7, 13, 19, 26, 29, 32, 33] where IF is applied to their objective functions, 11 case studies (MSE=1, MF=5, GFM=5) have been investigated. Table 7 shows the maximum and minimum values of effective parameters among all 11 case studies containing shear frames. Using values presented in Table 7 instead of minimum and maximum values of effective parameters in Eq. (3), $IF_{shear\ frame}$ is then given by:

$$IF_{shear\ frame}=\frac{N}{30}+\frac{1}{M}+\frac{1}{2}\times\left(\frac{\xi_f}{8}+\frac{\xi_\Phi}{3}\right)+\frac{D}{4} \tag{6}$$

Table 7: The maximum and minimum values of effective parameters in shear frame

N_{max}	M_{min}	ξ_{fmax} (%)	$\xi_{\Phi max}$ (%)	D_{max}
30	1	8	3	4

The chart presented in Fig. 3 indicates that the frequency of the papers with objective functions based on MF and GFM are equal to each other (45.5%), and the frequency of the papers based on MSE is considerably less than those of MF and GFM (9%). By putting N, M, ξ_f, ξ_Φ, and D in Eq. (6), the best objective functions among all shear frames based on MSE, MF, and GFM will be obtained. Results are given in Tables 8 through 10. Table 11 contains the minimum, mean, and maximum values of IF for each modal parameter in shear frames.

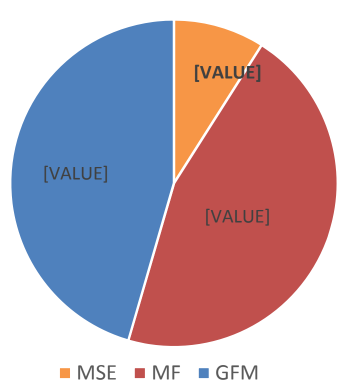

Figure 3. Frequency of examined modal parameters of shear frames

Table 8: Obtained values of $IF_{shear\ frame}$ in objective functions based on MSE

Article number	N	M	ξ_f (%)	ξ_Φ (%)	D	$\frac{N}{30}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{8}+\frac{\xi_\Phi}{3}\right)$	$\frac{D}{4}$	$IF_{shear\ frame}$
14	7	4	0.15	3	2	0.23	0.25	0.51	0.50	$IF_{max} = IF_{min}$ =1.49
$\overline{IF}_{MSE} = 1.49$										

Table 9: Obtained values of $IF_{shear\ frame}$ in objective functions based on MF

Article number	N	M	ξ_f (%)	ξ_Φ (%)	D	$\frac{N}{30}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{8}+\frac{\xi_\Phi}{3}\right)$	$\frac{D}{4}$	$IF_{shear\ frame}$
19	3	1	0	2	3	0.10	1.00	0.33	0.75	2.18
19	30	5	0	0	1	1.00	0.20	0.00	0.25	1.45
3	5	3	0	0	2	0.17	0.33	0.00	0.50	IF_{min} =1.00
4	25	1	5	0	4	0.83	1.00	0.31	1.00	IF_{max} =3.14
10	7	2	8	0	2	0.23	0.50	0.50	0.50	1.73
$\overline{IF}_{MF}$ =1.90										

Table 10: Obtained values of $IF_{shear\ frame}$ in objective functions based on GFM

Article number	N	M	ξ_f (%)	ξ_Φ (%)	D	$\frac{N}{30}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{8}+\frac{\xi_\Phi}{3}\right)$	$\frac{D}{4}$	$IF_{shear\ frame}$
22	5	1	2	1	2	0.17	1.00	0.29	0.50	1.96
22	10	2	5	0	2	0.33	0.50	0.31	0.50	IF_{min} =1.64
22	15	1	5	0	3	0.50	1.00	0.31	0.75	IF_{max} =2.56
25	10	1	5	0	3	0.33	1.00	0.31	0.75	2.39
26	5	1	8	0	2	0.17	1.00	0.50	0.50	2.17
$\overline{IF}_{GFM} = 2.14$										

Table 11: The minimum, mean, maximum values of $IF_{shear\ frame}$ for each modal parameter

	Modal parameters	IF_{min}	IF_{max}	$\overline{IF}_m$
Shear Frames	MSE	1.49	1.49	1.49
	MF	1.00	3.14	1.90
	GFM	1.64	2.56	2.14

3.3 Moment-resisting frames

15 case studies (MSE=5, MF=6, GFM=4) from 14 papers [5, 8, 10, 12-14, 18, 20-22, 27, 32, 33, 35] are investigated, applying IF to their objective functions. The maximum and minimum values of effective parameters among all 15 case studies are indicated in Table 12. IF_{MRF} is acquired by replacing the maximum and minimum values of effective parameters in Eq. (3) with values obtained from Table 12:

$$IF_{MRF} = \frac{N}{56} + \frac{1}{M} + \frac{1}{2} \times \left(\frac{\xi_f}{5} + \frac{\xi_\Phi}{8} \right) + \frac{D}{4} \tag{7}$$

Table 12: The maximum and minimum values of effective parameters in moment-resisting frame

N_{max}	M_{min}	ξ_{fmax} (%)	$\xi_{\Phi max}$ (%)	D_{max}
56	1	5	8	4

As illustrated in Fig. 4, frequency of studies with MF based objective functions (40%) are clearly more than those of MSE (33.3%) and GFM (26.7%). As indicated in Table 13 through 15, the best objective functions among all moment-resisting frames based on MSE, MF, and GFM are obtained from Eq. (7) by using N, M, ξ_f, ξ_Φ, and D. Table 16 contains minimum, mean and maximum values of IF_{MRF} for modal parameters in the moment-resisting frame.

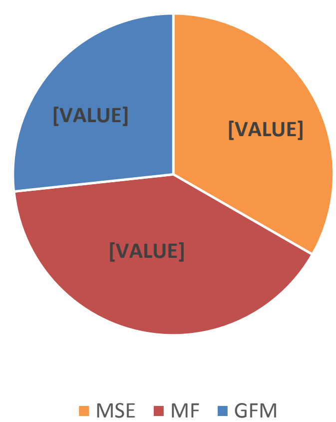

Figure 4. Frequency of examined modal parameters of the moment-resisting frames

Table 13: Obtained values of IF_{MRF} in objective functions based on MSE

Article number	N	M	ξ_f(%)	ξ_Φ(%)	D	$\frac{N}{56}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{5}+\frac{\xi_\Phi}{8}\right)$	$\frac{D}{4}$	IF_{MRF}
5	56	1	0	0	3	1.00	1.00	0.00	0.75	IF_{max} =2.75
16	56	5	0	0	4	1.00	0.20	0.00	1.00	2.20
2	33	1	0	5	3	0.59	1.00	0.31	0.75	2.65
9	18	6	0	0	2	0.32	0.17	0.00	0.50	IF_{min} =0.99
11	18	2	0	0	2	0.32	0.50	0.00	0.50	1.32
$\overline{IF}_{MSE}$ = 1.98										

Table 14: Obtained values of IF_{MRF} in objective functions based on MF

Article number	N	M	ξ_f(%)	ξ_Φ(%)	D	$\frac{N}{56}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{5}+\frac{\xi_\Phi}{8}\right)$	$\frac{D}{4}$	IF_{MRF}
20	9	1	5	0	3	0.16	1.00	0.50	0.75	2.41
13	4	10	1	1	3	0.80	0.10	0.16	0.75	IF_{min} =1.81
17	2	5	0.5	8	3	0.43	0.20	0.55	0.75	1.93
17	2	5	0.5	8	3	0.50	0.20	0.55	0.75	2.00
28	9	1	5	0	4	0.16	1.00	0.50	1.00	IF_{max} =2.66
10	1	4	3	3	3	0.27	0.25	0.49	0.75	1.76
$\overline{IF}_{MF}$ =2.10										

Table 15: Obtained values of IF_{MRF} in objective functions based on GFM

Article number	N	M	ξ_f(%)	ξ_Φ(%)	D	$\frac{N}{56}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{5}+\frac{\xi_\Phi}{8}\right)$	$\frac{D}{4}$	IF_{MRF}
7	21	8	0.5	2	4	0.38	0.13	0.18	1.00	1.69
15	12	6	5	0	3	0.21	0.17	0.50	0.75	IF_{min} =1.63
25	23	1	5	0	4	0.41	1.00	0.50	1.00	IF_{max} =2.91
26	19	1	5	0	3	0.34	1.00	0.50	0.75	2.59
$\overline{IF}_{GFM}$ = 2.21										

Table 16: The minimum, mean, maximum values of IF_{MRF} for each modal parameter

	Modal parameters	IF_{min}	IF_{max}	$\overline{IF}_m$
Moment-resisting frames	MSE	0.99	2.75	1.98
	MF	1.81	2.66	2.10
	GFM	1.63	2.91	2.21

3.4 Planar trusses

In 17 planar truss case studies (MSE=8, MF=5, GFM=4) from 15 papers [8-10, 13-15, 20-22, 25, 27, 30, 32-34] using IF in their objective functions are explored. The maximum and minimum values of effective parameters among all planar trusses are presented in Table 17. By replacing the maximum and minimum values of effective parameters with the above parameters in Table 17, the modified Eq. (3) is achieved. Thus, $IF_{2D\text{-}Truss}$ is obtained as follows:

$$IF_{2D-Truss}=\frac{N}{78}+\frac{1}{M}+\frac{1}{2}\times\left(\frac{\xi_f}{5}+\frac{\xi_\Phi}{10}\right)+\frac{D}{5} \tag{8}$$

Table 17: The maximum and minimum values of effective parameters in planar truss

N_{max}	M_{min}	ξ_{fmax} (%)	$\xi_{\Phi max}$ (%)	D_{max}
78	1	5	10	5

It should be noted that among investigated moment-resisting frames objective functions based on MSE (47.1%) appear more frequently compared to the ones based on MF (29.4%) and GFM (23.5%) as depicted in Fig. 5. As demonstrated in Table 18 through Table 20, Eq. (8) can be modified by using N, M, ξ_f, ξ_Φ, and D, in order to get the best objective functions among all planar trusses based on MSE, MF, and GFM. Furthermore, the minimum, mean, and maximum values of $IF_{2D\text{-}Truss}$ for each modal parameter in planar trusses are presented in Table 21.

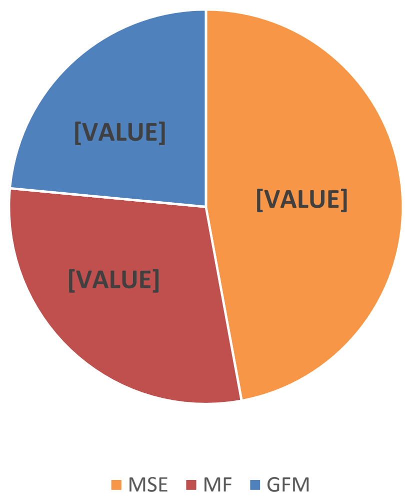

Figure 5. Frequency of examined modal parameters of the planar trusses

Table 18: Obtained values of $IF_{2D\text{-}Truss}$ in objective functions based on MSE

Article number	N	M	$\xi_f(\%)$	$\xi_\Phi(\%)$	D	$\frac{N}{78}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{5}+\frac{\xi_\Phi}{10}\right)$	$\frac{D}{5}$	$IF_{2D-Truss}$
5	31	3	0	3	2	0.40	0.33	0.15	0.40	1.28
16	37	5	0	0	5	0.47	0.20	0.00	1.00	1.67
6	10	4	0	0	2	0.13	0.25	0.00	0.40	IF_{min} =0.78
6	31	10	0	0	2	0.40	0.10	0.00	0.40	0.90
6	47	10	0	0	2	0.60	0.10	0.00	0.40	1.10
27	19	5	2	10	5	0.24	0.20	0.70	1.00	IF_{max} =2.14
11	78	5	0	5	3	1.00	0.20	0.25	0.60	2.05
12	30	3	0	3	3	0.38	0.33	0.15	0.60	1.46
$\overline{IF}_{MSE}$ = 1.42										

Table 19: Obtained values of $IF_{2D\text{-}Truss}$ in objective functions based on MF

Article number	N	M	$\xi_f(\%)$	$\xi_\Phi(\%)$	D	$\frac{N}{78}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{5}+\frac{\xi_\Phi}{10}\right)$	$\frac{D}{5}$	$IF_{2D-Truss}$
18	31	4	0	3	2	0.40	0.25	0.15	0.40	1.20
20	13	1	5	0	2	0.17	1.00	0.50	0.40	IF_{max} =2.07
17	21	5	0.5	8	3	0.27	0.20	0.45	0.60	1.52
23	31	5	0	0	1	0.40	0.20	0.00	0.20	IF_{min} =0.80
10	13	3	5	0	4	0.17	0.33	0.50	0.80	1.80
$\overline{IF}_{MF}$ =1.48										

Table 20: Obtained values of $IF_{2D\text{-}Truss}$ in objective functions based on GFM

Article number	N	M	$\xi_f(\%)$	$\xi_\Phi(\%)$	D	$\frac{N}{78}$	$\frac{1}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{5}+\frac{\xi_\Phi}{10}\right)$	$\frac{D}{5}$	$IF_{2D-Truss}$
7	25	8	0.5	2	3	0.32	0.13	0.15	0.60	IF_{min} =1.20
15	21	3	5	0	3	0.27	0.33	0.50	0.60	1.70
25	29	1	5	0	3	0.37	1.00	0.50	0.60	IF_{max} =2.47
26	15	1	3	0	2	0.19	1.00	0.30	0.40	1.89
$\overline{IF}_{GFM}$ =1.82										

Table 21: The minimum, mean, maximum values of $IF_{2D\text{-}Truss}$ for each modal parameter

	Modal parameters	IF_{min}	IF_{max}	$\overline{IF}_m$
Planar trusses	MSE	0.78	2.14	1.42
	MF	0.80	2.07	1.48
	GFM	1.20	2.47	1.82

3.5 Spatial trusses

In these structures, 5 case studies from 5 different studies (MSE=3, MF=2, GFM=0) are investigated [4, 8, 15, 18, 25]. The maximum and minimum values of effective parameters among all spatial trusses are shown in Table 22. $IF_{3D\text{-}Truss}$ is obtained similar to the abovementioned cases by using Eq. (3) and the values demonstrated in Table 22 as:

$$IF_{3D-Truss} = \frac{N}{312} + \frac{3}{M} + \frac{1}{2}\times\left(\frac{\xi_f}{1} + \frac{\xi_\Phi}{5}\right) + \frac{D}{4} \tag{9}$$

Table 22: The maximum and minimum values of effective parameters in spatial truss

N_{max}	M_{min}	ξ_{fmax} (%)	$\xi_{\Phi max}$ (%)	D_{max}
312	3	1	5	4

By investigating papers that had spatial truss as a numerical example, the majority of them have used objective functions based on MSE (60%), while the rest have applied MF (40%) with no study using GFM (0%) as shown in Fig. 6. In addition, similar to previous cases, the best objective functions among all spatial trusses based on MSE, MF, and GFM are obtained by putting N, M, ξ_f, ξ_Φ, and D in Eq. (9). Table 23 and Table 24 indicates the acquired results. Finally, as presented in Table 25, the minimum, mean, and maximum values of $IF_{3D\text{-}Truss}$ for each modal parameter in spatial trusses are obtained.

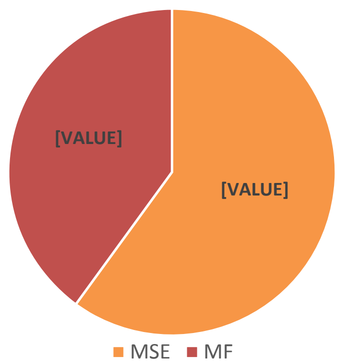

Figure 6. Frequency of examined modal parameters of the spatial trusses

Table 23: Obtained values of $IF_{3D\text{-}Truss}$ in objective functions based on MSE

Article number	N	M	$\xi_f(\%)$	$\xi_\Phi(\%)$	D	$\frac{N}{312}$	$\frac{3}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{1}+\frac{\xi_\Phi}{5}\right)$	$\frac{D}{4}$	$IF_{3D-Truss}$
5	25	3	0	3	3	0.08	1.00	0.30	0.75	IF_{min} =2.13
1	31	5	0	5	4	1.00	0.60	0.50	1.00	IF_{max} =3.10
12	50	3	0	3	3	0.16	1.00	0.30	0.75	2.21
$\overline{IF}_{MSE}$ =2.48										

Table 24: Obtained values of $IF_{3D\text{-}Truss}$ in objective functions based on MF

Article number	N	M	$\xi_f(\%)$	$\xi_\Phi(\%)$	D	$\frac{N}{312}$	$\frac{3}{M}$	$\frac{1}{2}\times\left(\frac{\xi_f}{1}+\frac{\xi_\Phi}{5}\right)$	$\frac{D}{4}$	$IF_{3D-Truss}$
18	52	5	0	3	4	0.17	0.60	0.30	1.00	IF_{min} =2.07
13	52	1	1	3	4	0.17	0.30	0.80	1.00	IF_{max} =2.27
$\overline{IF}_{MF}$ = 2.17										

Table 25: The minimum, mean, maximum values of $IF_{3D\text{-}Truss}$ for each modal parameter

	Modal parameters	IF_{min}	IF_{max}	$\overline{IF}_m$
Spatial truss	MSE	2.13	3.10	2.48
	MF	2.07	2.27	2.17

Finally, all obtained results based on the aforementioned numerical studies are summarized in Table 26.

Table 26: Results of paper's objective functions based on MSE, MF, and GFM in the examined structures

structure	Modal parameter	Number of case studies	IF_{min}	IF_{max}	$\overline{IF}_m$	Frequency content (%)	IF of best objective function
Beam-like structures	MSE	6	0.47	2.23	1.28	46.20	2.23
	MF	4	0.42	2.10	1.56	30.80	
	GFM	3	1.63	1.80	1.72	23.00	
Shear frames	MSE	1	1.49	1.49	1.49	9.00	3.14
	MF	5	1.00	3.14	1.90	45.50	
	GFM	5	1.64	2.56	2.14	45.50	
Moment-resisting frames	MSE	5	0.99	2.75	1.98	33.30	2.91
	MF	6	1.81	2.66	2.10	40.00	
	GFM	4	1.63	2.91	2.21	26.70	
Planar trusses	MSE	8	0.78	2.14	1.42	47.10	2.47
	MF	5	0.80	2.07	1.48	29.40	
	GFM	4	1.20	2.47	1.82	23.50	
Spatial trusses	MSE	3	2.13	3.10	2.48	60	3.10
	MF	2	2.07	2.27	2.17	40	

As can be shown in Table 26, it can be concluded that the best IF in beam-type structures and spatial truss is related to MSE. Additionally, in moment-resisting frames and planar trusses the maximum value of IF belongs to GFM, and just in shear frames, it relates to MF. Consequently, according to the obtained results at four different types of investigated structures out of five, the best IF is related to MSE and GFM. In other words, investigating numerous papers reveal that defining objective functions based on the combination of GFM and MSE can enhance the efficiency and robustness of detecting damage more than either MSE, GFM, or even MF in future studies.

4. CONCLUSIONS

The fundamental aim of SHM is to gather information about damages in the structure. Most of the problems related to damage detection can be approached as an inverse problem using an objective function. In this study, related researches using modal strain energy and flexibility methods in their objective functions are reviewed. The effectiveness of these functions is investigated and depicted in a structured way using an efficient indicator named IF. Although the results in all structures indicate that objective functions based on GFM have been used less frequently than MF and MSE, the mean value of IF for GFM is more than those of MF and MSE. Moreover, spatial truss has not been used in the objective function based on GFM yet. On the other hand, the results illustrate that objective functions depending on MSE occur more often than those based on MF and GFM among all investigated structures meaning that it has responded and detected damage in numerous structures. Additionally, since MF and GFM are similar and GFM is the modified version of MF, by combining objective functions based on GFM and MSE, best objective function can be obtained with the highest value of IF for future researches.

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