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DESIGN OPTIMIZATION OF MOMENT FRAME STRUCTURES BASED ON NATURAL FREQUENCY CONSTRAINTS USING THE ADAPTIVE CHARGED SYSTEM SEARCH ALGORITHM

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ABSTRACT

Natural frequencies of a structure give useful information about the structural response to dynamic loading. These frequencies should be far enough from the critical frequency range of dynamic excitations like earthquakes in order to prevent the resonance phenomenon sufficiently. Although there are many investigations on optimization of truss structures subjected to frequency constraints, just a few studies have been considered for optimal design of frame structures under these constraints. In this paper, a recently proposed metaheuristic algorithm called Adaptive Charged System Search (ACSS) is applied to optimal design of steel frame structures considering the frequency constraints. Benchmark design examples are solved with the ACSS, and optimization results are illustrated in terms of some statistical indices, convergence history and solution quality. The design examples include three planar steel frames with small to large number of design variables. Results show that the ACSS outperforms the charged system search algorithm in this sizing optimization problem.

Keywords: planar frames; frequency constraints; structural optimization; adaptive charged system search (ACSS); moment-resisting frames.

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1. INTRODUCTION

Design optimization of mechanical and structural systems subjected to frequency constraints has been investigated by many researchers [1-4]. The importance of this investigation is due to the fact that the dynamic response of a structure can be expressed in terms of its natural frequencies and modal shapes, and hence one can design a structure with some natural frequency constraints for avoiding the resonance phenomenon in some vibration modes.

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Optimal structural design for frequency constraints is a highly nonlinear problem which is usually difficult to be solved by a gradient-based optimizer due to the change of vibration modes with modifying sizing variables. The vibration mode variation motivates the convergence difficulties for an optimization algorithm. Also, another concern is that some optimally designed structures represent the repeated eigenvalues increasing the complexity of the problem.

Nowadays, metaheuristic algorithms have been applied to various engineering optimization problems [3, 5-8]. These algorithms require neither gradient information of objective function nor those of constraints and have been inspired by physical or natural phenomena. Many researchers have focused on structural optimization with metaheuristic methods. A part of these studies conducted the optimal design of structures under different constraints considering gravity, wind and earthquake loadings [9-15]. On the other hand, numerous studies were carried out for optimal design of truss structures based on frequency constraints for which metaheuristic methods consisting of particle swarm optimization [16], adaptive hybrid evolutionary firefly [17], symbiotic organisms search [18], vibrating particles system [19], collaborative optimization strategy [20], hybridized optimization approaches [21], hybridized optimality criteria and genetic algorithm [22], and enhanced differential evolution [23] were applied. Nevertheless, few researchers have optimized the design of frame structures with frequency constraints. Clearly, optimal design of frame structures is more complex than that of truss structures because the number of degrees of freedom for the frames is higher, leading to larger eigenproblem being more sensitive to the alteration of sizing variables in an optimization problem.

McGee and Phan [24, 25] performed the optimal design of planar and space frame structures with an efficient optimality criteria method in which an iterative method according to the Karush-Kuhn-Tucker optimality condition was used. Salajegheh [26] proposed the response approximation and optimality criteria methods for design optimization of grid and frame structures subjected to frequency constraints such that the desired derivatives of the functions were computed by a semi-analytical approach in order to reduce the number of structural analyses. Also, Salajegheh [27, 28] derived two-point and three-point approximation of the Rayleigh quotient for estimation of the eigenvalues required for optimizing frames subjected to frequency limitations. Although metaheuristic algorithms have widely been used for optimizing truss structures under frequency constraints, only one study has been found for metaheuristic-based design optimization of frame structures under these constraints [29].

This study applies the adaptive charged system search (ACSS) for optimal design of steel frame structures with multiple frequency limitations. In order to show the capability of the ACSS, three planar frames including two-story, seven-story and ten-story structures are considered as design examples. Optimization results are reported using statistical analysis, convergence curves and optimized design variables.

2. THE OPTIMIZATION PROBLEM DESCRIPTION

The sizing optimization problem of a structure with frequency constraints is expressed as follows:

$$\begin{aligned}&\text{Minimize} && w(\mathbf{x})\\ &\text{subject to} && g_j(\mathbf{x}) \le 0\\ &\mathbf{x} = [x_1, x_2, x_3, ..., x_{nv}]^{\mathrm{T}}\\ &x_i \in \Re\end{aligned} \tag{1}$$

in which **x** denotes the vector of design variables including the cross-sectional areas, and each design variable is limited by its lower bound ($x_{\min}$) and upper bound ($x_{\max}$). Also, nv represents the number of design variables. $w(\mathbf{x})$ and $g_j(\mathbf{x})$ denote the weight of the structure and the jth constraint, respectively. For constraint handling, the penalty approach is used here, which transforms a constrained optimization problem into an unconstrained optimization problem as follows:

$$\text{Minimize} \qquad f_{obj}(\mathbf{x}) = w(\mathbf{x}) \times p(\mathbf{x}) \tag{2}$$

where $f_{obj}(\mathbf{x})$ represents the objective function (i.e., the penalized structural weight), and $p(\mathbf{x})$ shows the penalty function. The weight and the penalty function are defined by:

$$w(\mathbf{x}) = \sum_{i=1}^{nv} \gamma_i \cdot x_i \cdot l_i \tag{3}$$

$$p(\mathbf{x}) = (1 + \kappa_1 \cdot \nu)^{\kappa_2} \;,\; \nu = \sum_{j=1}^{n} \max\{0, \nu_j\} \tag{4}$$

where l_i and γ_i are the length and the material density of the ith design variable, respectively; the sum of the violated constraints is indicated by ν ; and ν_j denotes the amount of the jth constraint violation. For the penalty function, κ_1 is often chosen as unity and κ_2 is an increasing function of iteration. Also, in order to check whether or not the frequency constraints are violated, an eigenvalue analysis should be carried out for every objective function evaluation. A frequency constraint is expressed as:

$$g_j(\mathbf{x}) = \frac{f_j^*}{f_j} - 1 \le 0 \tag{5}$$

in which f_j^* indicates the prescribed lower bound of frequency for jth mode and f_j is the frequency corresponding to the solution vector, **x**, for the *j*th mode.

Unlike the analysis of trusses, cross-sectional properties of frame members are not defined only by the cross-sectional areas. Thus, the remaining cross-sectional properties are expressed in terms of the primary variable (cross-sectional area), x_i, so that continuous form of the optimization problem can be maintained. Empirical relationships between the primary and the secondary variables for prevalent wide-flange steel sections in the American Institute of Steel Construction (AISC) Manual [30] have been derived as [24]:

$$I_{zi} = \begin{cases} 4.6248\, x_i^2, & 0 < x_i < 44.2 \text{ in}^2 \\ 256\, x_i - 2300, & 44.2 < x_i < 88.3 \text{ in}^2 \end{cases} \tag{6}$$

in which the secondary variable is I_{zi} denoting the principal moment of inertia about major axis for the *i*th member. The primary variables are chosen as optimization variables, by which the number of design variables is reduced because the secondary variables are obtained from the primary ones.

3. EIGENVALUE ANALYSIS OF PLANAR FRAME STRUCTURES

This section provides the mass and stiffness matrices of planer frame structures for the eigenvalue analysis required for the optimization problem. The consistent mass matrix of a frame element is expressed as [29]:

$$\mathbf{m}_e = \frac{\rho A L}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ & 156 & 22L & 0 & 54 & -13L \\ & & 4L^2 & 0 & 13L & -3L^2 \\ & & & 140 & 0 & 0 \\ & symmetric & & & 156 & -22L \\ & & & & & 4L^2 \end{bmatrix} \tag{7}$$

and stiffness matrix of a frame element is obtained as [29]:

$$\mathbf{k}_e = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & symmetric & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix} \tag{8}$$

with E, ρ and L being elastic modulus, mass density and length of an element with cross-sectional properties of A and I which are the area and the moment of inertia, respectively. Transformation of these element matrices from the local coordinate system to the global coordinate system and the assembly procedure result into the following eigenvalue problem:

$$\mathbf{K}\boldsymbol{\varphi}_i = \omega_i^2 \mathbf{M}\boldsymbol{\varphi}_i \tag{9}$$

where ω_i and $\boldsymbol{\varphi}_i$ are the circular frequency and mode shape vector for the *i*th mode of vibration, respectively. **K** and **M** denote the assembled stiffness and mass matrices of the structure. In this study, natural frequencies of a few modes corresponding to the smallest eigenvalues are needed, which can be calculated as per Refs. [31, 32].

4. THE ADAPTIVE CHARGED SYSTEM SEARCH

Recently, the adaptive charged system search (ACSS) algorithm was developed by Zakian and Kaveh [5]. The ACSS is an improved variant of the charged system search (CSS) algorithm. Like the CSS [33, 34], the ACSS is a population-based algorithm consisting of charged particles (CPs). Each particle (or agent) is defined as a sphere with uniform charge density and radius of *a*. Every CP is affected by the force field of particles. The resultant force is calculated with the electrostatics laws, and the quality of the movement is based on Newtonian mechanics. A good CP must impose a larger force than that of a bad CP.

The ACSS uses two improvements for the CSS, including a modified initialization and a modified random walk [5]. The step-by-step procedure of the ACSS algorithm is summarized as follows:

Step 1: Initialization.

Initial positions of CPs are defined randomly in the search space, while the initial velocities of CPs are chosen to be zero. The values of objective function for the CPs are calculated and are sorted in an ascending order. The best CP among the entire set of CPs is selected as x_{best} and its corresponding objective function value is f_{best}. Similarly, the objective function value corresponding to the worst CP is taken as f_{worst}.

In contrast to the CSS which uses one search space for initialization, the ACSS uses four search spaces [5]. In other words, three new spaces are added to that of the CSS. In the first space, the initialization of the ACSS is similar to the CSS. In the second space, the opposition-based learning (OBL) concept is employed for the initialization step. The OBL is a concept in soft computing for improving the convergence rate of optimizers [35]. The OBL uses a population with its opposite counterpart to consider better potential solutions. Previous investigations have shown that using the OBL increases the probability of finding global optimum [35, 36]. As a simple definition of an opposite number, assume x as a real number within [a, b] then its opposite number is equal to $a+b-x$. In the third space and the fourth space, the random numbers are initialized using the lower bound and upper bound subdomains. This is because optimal solutions have usually a tendency to be close to the boundaries of domain, and hence one can divide a domain to upper bound and lower bound subdomains to take this point into account. Consequently, four spaces are introduced in the initialization part of the ACSS as given by [5]:

- Space 1 (ordinary; like the CSS):

$$x_{i,j}^{Initial_1} = x_{i,\min} + rand \cdot (x_{i,\max} - x_{i,\min}), \qquad i = 1,2,...,nv \tag{10}$$

- Space 2 (based on the OBL):

$$x_{i,j}^{Initial_2} = x_{i,\max} + x_{i,\min} - x_{i,j}^{Initial_1}, \qquad i = 1,2,...,nv \tag{11}$$

- Space 3 (lower bound):

$$x_{i,j}^{Initial_3} = x_{i,\min} + rand \cdot (\frac{x_{i,\min} + x_{i,\max}}{2} - x_{i,\min}), \quad i = 1,2,...,nv \tag{12}$$

- Space 4 (upper bound):

$$x_{i,j}^{Initial_4} = \frac{x_{i,\min} + x_{i,\max}}{2} + rand \cdot (x_{i,\max} - \frac{x_{i,\min} + x_{i,\max}}{2}), \qquad i = 1,2,...,nv \tag{13}$$

After initializing the design variables, best solutions among the four spaces are stored in a charge memory (CM) without any change in the size of CM with respect to that of the CSS. No additional computational efforts are imposed to the algorithm during the iterative process. In other words, the four spaces are utilized only for the initialization step. The CM is a matrix wherein a number of the best CPs and their corresponding values of the objective function are stored. Here, *rand* is a uniformly distributed random number within [0,1].

Step 2: Force determination.

The force vector for a CP is calculated as:

$$F_j = q_j \sum_{i,i \neq j} (\frac{q_i}{a^3} r_{ij} .i_1 + \frac{q_i}{r_{ij}^2} .i_2) p_{ij} (x_i - x_j) \qquad \begin{cases} j = 1,2,...,n \\ i_1 = 1, \;\; i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, \;\; i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{14}$$

in which F_j denotes the resultant force acting on the jth CP and n is the total number of CPs. The charge magnitude of each CP, q_i, is obtained according to its solution quality, that is:

$$q_i = \frac{f_{obj}(i) - f_{worst}}{f_{best} - f_{worst}} \qquad i = 1,2,...,n \tag{15}$$

In addition, the distance r_{ij} between two particles is computed by:

$$r_{ij} = \frac{\left\| x_i - x_j \right\|}{\left\| (x_i + x_j)/2 - x_{best} \right\| + \varepsilon} \tag{16}$$

where the positions of the *i*th and *j*th CPs are based on x_i and x_j, respectively. In order to avoid the singularity, an infinitesimally positive real number (ε) is added to the denominator. Here, p_{ij} is the probability of each particle movement towards the others, that is:

$$p_{ij} = \begin{cases} 1 & \dfrac{f_{obj}(i) - f_{best}}{f_{obj}(j) - f_{obj}(i)} > rand \;\; or \;\; f_{obj}(j) > f_{obj}(i) \\ 0 & else \end{cases} \tag{17}$$

Radius of a charged sphere is obtained using the following relation:

$$a = c_0 \times \max\left\{ (x_{i,\max} - x_{i,\min}) \right\}; \qquad i = 1, 2, ..., nv \tag{18}$$

with c_0 being a constant coefficient which is close to 0.005 here.

Step 3: Solution and updating procedure.

Each CP moves to its new position based on the resultant force of the CP, old velocity and the old position. In the ACSS, Levy flight algorithm is used to enhance the random exploration [5]. Levy flight is an efficient random walk which has been implemented in some optimization algorithms [37, 38]. Levy motion is a non-Gaussian random process whose random walks are based on Levy distribution as a power law formula. In mathematical representation, a simple version of Levy distribution can be written as:

$$L(s, \gamma, \mu) = \begin{cases} \sqrt{\dfrac{\gamma}{2\pi}} \exp\left[-\dfrac{\gamma}{2(s-\mu)} \right] \dfrac{1}{(s-\mu)^{3/2}} & 0 < \mu < s < \infty, \\ 0 & s \le 0 \end{cases} \tag{19}$$

in which μ parameter is a shift parameter; $\gamma > 0$ parameter is a scale factor; and α is the skewness parameter within [-1,1].

Here, the influence of the local best solution, x_{best}, is formulated through Levy flight. Therefore, the ACSS uses the following equation for determining the new position of particles [5]:

$$x_{j,new} = rand_{j1} \cdot F_j + rand_{j2} \cdot k_v \cdot v_{j,old} + 0.01 \frac{u}{|v|^{2/3}} (x_{best} - x_{j,old}) + x_{j,old} \tag{20}$$

where u and v are randomly selected numbers with normal distribution; $rand_{j1}$ and $rand_{j2}$ are random numbers uniformly distributed in [0,1]. The third term on the right-hand side of Eq. (20) represents the Levy flight contribution.

Also, the new velocity is determined by:

$$v_{j,new} = x_{j,new} - x_{j,old} \tag{21}$$

It should be noted that the ACSS does not use the acceleration coefficient (k_a) [5], but it uses k_v with c_v being equal to or less than those of the CSS. k_v is a decreasing function known as the velocity coefficient to stabilize the effect of the previous velocity and to control the exploration, as given by:

$$k_v = c_v \times [1 - (\frac{iter}{iter_{\max}})] \tag{22}$$

in which $iter$ is the current iteration number and $iter_{max}$ is the maximum number of iterations. c_v is a constant value adjusted based on the optimization problem.

Similar to the CSS, when a new CP is outside the allowable search space during the updating process, a harmony search-based approach can be used to limit its position to the allowable search space [33]. This strategy permits any component of the solution vector violating the variable boundaries to be regenerated from the CM or from a randomly selected value belonging to the allowable range of variables. In addition, if there are some new CPs better than the worst ones in the CM, then the worst CPs in the CM are substituted by the better solution vectors (that is, better CPs).

Step 4: Stopping criterion.

Steps 2 and 3 must be repeated until meeting a prescribed stopping criterion. In this study, the maximum number of iterations is chosen as the stopping criterion.

5. DESIGN EXAMPLES

In this section, optimal structural design of three planar frames under frequency constraints is carried out in order to assess the capability of the ACSS algorithm. Weight density and elasticity modulus of the steel material for these frames are taken as 0.28 lb/in^3 and 30 Msi, respectively [29]. Mass source of these structures includes both structural mass and non-structural mass. The consistent mass matrix is used for defining the structural mass, whereas the lumped mass matrix is utilized for defining the non-structural mass. For the non-structural mass, the lumped mass corresponding to transitional degrees of freedom at horizontal direction is considered. Number of particles used in examples 1, 2 and 3 are selected as 20, 30 and 40, respectively. The performance of the ACSS is compared to the solutions of other metaheuristic algorithms in Ref. [29], those which used the same number of structural analyses. Due to the inherent randomness of the algorithms, optimization results are based on 20 independent runs.

5.1 Planar two-story frame

The first example is a benchmark frame (shown in Fig. 1) with cross-sectional areas ranging from 3 in^2 to 88.28 in^2 [29]. The first three natural frequencies of the structure are constrained such that $f_1 \geq 5$ Hz, $f_2 \geq 18$ Hz and $f_3 \geq 35$ Hz. Non-structural uniform weight of 10 lb/in is loaded on the horizontal members. The gradient-based methods achieved the optimal weights of 3280.24 lb [25] and 3267.93 lb [27] for this structure. Also, Table 1 shows the results of other metaheuristic algorithms (obtained in Ref. [29]) including particle swarm optimization (PSO) [39], grey wolf optimizer (GWO) [40], improved grey wolf optimizer (IGWO) [14] and the CSS [33]. Although the ACSS gives better solutions than the GWO and the CSS, the PSO and the IGWO provide slightly smaller structural weight. Average and best convergence histories of the ACSS are drawn in Fig. 2. Based on Table 2, the constraint of the first mode is slightly violated by the ACSS but it is negligible compared to the results of other algorithms.

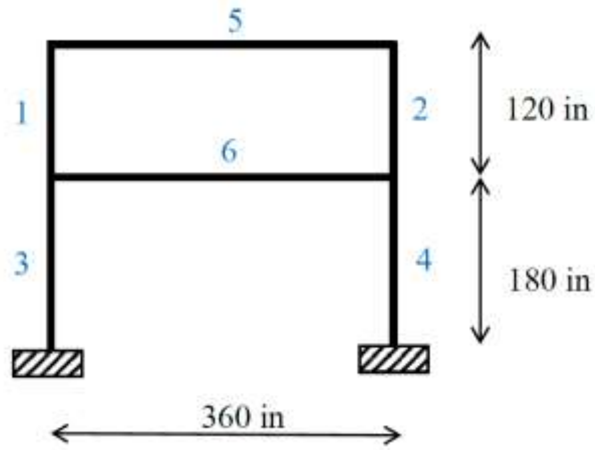

Figure 1. Geometry and member numbering of the planar two-story frame

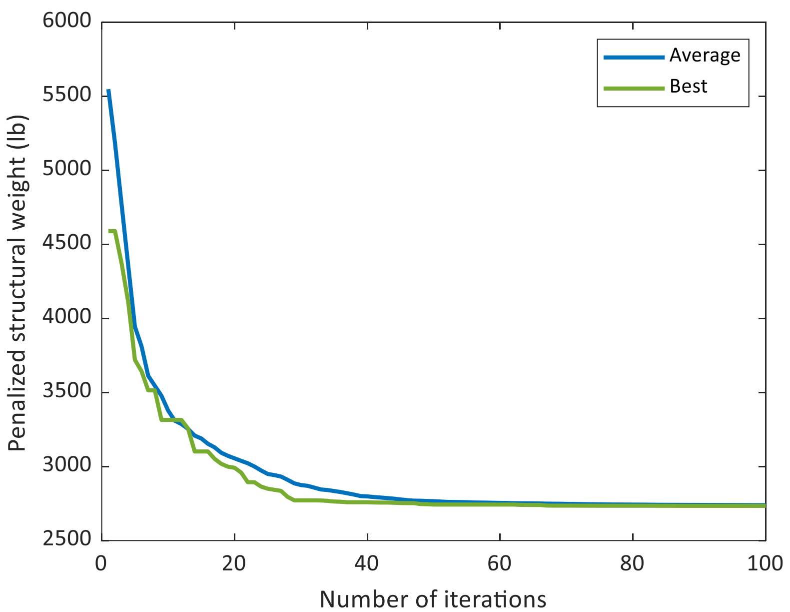

Figure 2. Convergence curves of the ACSS corresponding to the best result and the average of results obtained from 20 runs for the planar two-story frame

Table 1: Optimized designs obtained for the planar two-story frame; design variables are cross-sectional areas (in^2)

Variable no.	PSO [29]	CSS [29]	GWO [29]	IGWO [29]	ACSS (present study)
1	10.4830	3.7755	3.8064	10.7295	10.5348
2	3.7418	10.4271	10.5934	3.7342	3.7434
3	29.7480	3.0065	3.0000	29.5943	29.7063
4	3.0000	29.7753	29.7558	3.0002	3.0048
5	3.0000	3.0017	3.0225	3.0011	3.0009
6	3.0000	3.0034	3.0000	3.0000	3.0063
Best weight (lb)	2733.2520	2734.7324	2741.7958	2733.6560	2733.9139
Average weight (lb)	2795.2559	2778.2240	2795.5196	2736.7087	2738.7826
Standard deviation weight (lb)	126.1530	145.6291	27.1642	4.9400	2.9231

Table 2: Natural frequencies (Hz) computed for optimized designs of the planar two-story frame

Mode no.	PSO [29]	CSS [29]	GWO [29]	IGWO [29]	ACSS (present study)
1	5.0000	5.0002	5.0122	4.9999	4.9998
2	18.7548	18.7198	18.8507	18.9200	18.7907
3	35.0011	35.0113	35.1406	35.0068	35.0207

5.2 Planar seven-story frame

Here, a seven-story frame with 21 members is considered, as indicated in Fig. 3. Lower and upper bounds of cross-sectional areas are limited to 7.9187 in^2 and 88.28 in^2, respectively. Only the fundamental frequency of this structure is constrained to 1.6234 Hz [29]. Definition of non-structural mass is similar to the previous example. McGee and Phan [25] optimized this structure with the optimality criteria method and the obtained weight was 16537 lb. Here, Table 3 shows that the structure optimized by the ACSS is lighter than the structure optimized by the CSS, the GWO and the PSO. Natural frequencies of the first three modes are listed in Table 4 showing that the equality constraint of the first mode is satisfied. Furthermore, the average and the best convergence histories of the ACSS are illustrated in Fig. 4.

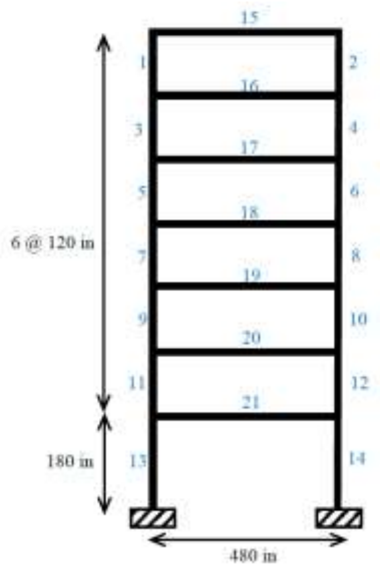

Figure 3. Geometry and member numbering of the planar seven-story frame

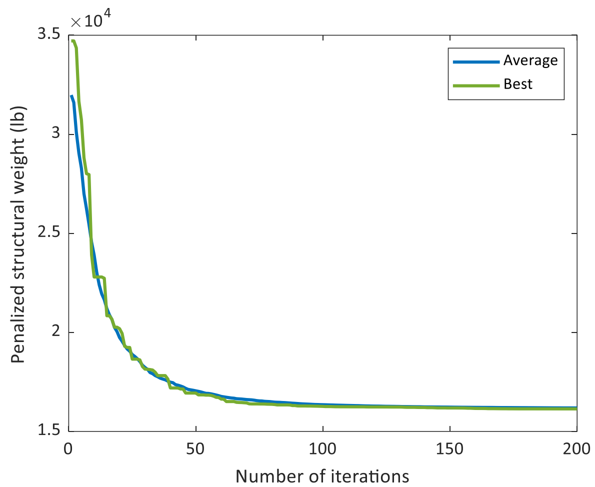

Figure 4. Convergence curves of the ACSS corresponding to the best result and the average of results obtained from 20 runs for the planar seven-story frame

Table 3: Optimized designs obtained for the planar seven-story frame; design variables are cross-sectional areas (in^2)

Variable no.	PSO [29]	CSS [29]	GWO [29]	IGWO [29]	ACSS (present study)
1	7.9187	7.9702	7.9187	7.9202	7.9215
2	7.9187	7.9540	7.9187	7.9187	7.9293
3	7.9187	7.9511	7.9187	8.0989	7.9938
4	7.9187	7.9399	7.9187	7.9374	7.9446
5	7.9187	8.1593	7.9187	7.9766	7.9331

6	15.9691	8.0299	8.8018	8.7332	7.9292
7	7.9187	8.0136	7.9187	9.7156	8.2603
8	11.1336	9.1955	14.1927	8.0608	10.0747
9	26.7111	11.8688	7.9187	8.4559	10.0165
10	7.9187	8.0259	7.9994	10.0986	8.0564
11	44.1940	8.0093	14.3966	14.2558	7.9940
12	7.9187	13.4150	10.5120	7.9231	13.9082
13	71.5218	7.9915	8.9831	41.5235	8.0281
14	7.9187	41.8599	46.1671	7.9187	40.0801
15	7.9187	7.9217	7.9187	7.9199	7.9442
16	7.9187	7.9261	7.9187	7.9187	7.9630
17	16.3990	8.6765	7.9187	9.2514	9.9582
18	7.9187	13.1870	14.7851	12.8494	12.9320
19	7.9187	14.1600	12.6396	14.8597	14.5069
20	7.9187	14.8598	16.0874	13.9370	14.2683
21	7.9187	8.0487	7.9187	7.9187	7.9379
Best weight (lb)	18015.0718	16142.3968	16625.4825	16123.8870	16133.5758
Average weight (lb)	21748.0732	16193.1860	16845.6255	16148.8398	16185.9713
Standard deviation weight (lb)	3144.8827	45.6982	151.1948	20.6601	36.6943

Table 4: Natural frequencies (Hz) computed for optimized designs of the planar seven-story frame

Mode no.	PSO [29]	CSS [29]	GWO [29]	IGWO [29]	ACSS (present study)
1	1.6234	1.6234	1.6282	1.6234	1.6233
2	4.9392	4.2478	4.2784	4.2616	4.2788
3	8.6486	8.1862	8.4829	8.2172	8.1359

5.3 Planar ten-story frame

In the last example, a ten-story frame with 70 members shown in Fig. 5 is optimized by the ACSS. The definition of non-structural mass is the same as that of the first example. Cross-sectional area of each member is taken as a design variable ranging from 3 in^2 to 88.28 in^2 [29]. The first three natural frequencies should be limited as follows: $f_1 \geq 2$ Hz, $f_2 \geq 7$ Hz and $f_3 \geq 15$ Hz. This structure was also studied by McGee and Phan [25] who obtained an optimum weight of 131648.5 lb. In this study, the ACSS outperforms the CSS, the GWO and the PSO but the solution of the IGWO is slightly better. However, the ACSS gives a relatively small standard deviation of the structural weight, which illustrates the suitable stability of the algorithm considering the inherent randomness. Convergence curves of the ACSS are shown in Fig. 6, and Tables 5 and 6 report the solution details. Also, natural frequencies calculated for the optimal solution are indicated in Table 7 demonstrating no violation of constraints.

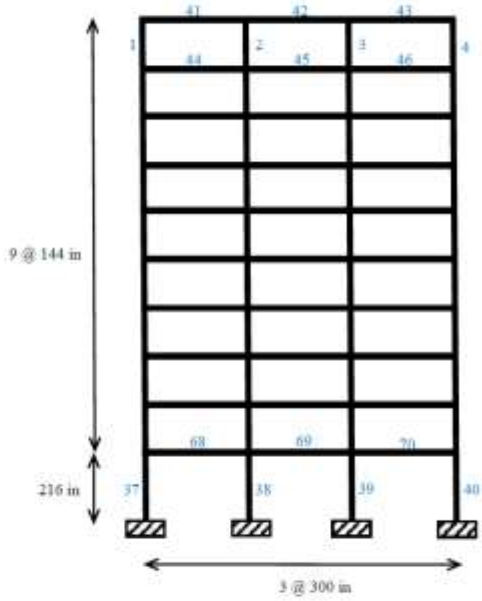

Figure 5. Geometry and member numbering of the planar ten-story frame

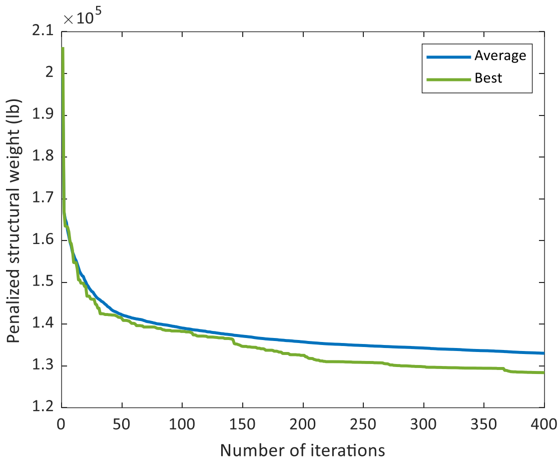

Figure 6. Convergence curves of the ACSS corresponding to the best result and the average of results obtained from 20 runs for the planar ten-story frame

Table 5: Optimized cross-sectional areas (in^2) obtained for the planar ten-story frame

PSO [29]				CSS [29]				GWO [29]				IGWO [29]				ACSS (present study)			
1	29.5197	36	33.6549	1	9.0595	36	37.4742	1	3.0000	36	14.8100	1	4.6338	36	41.9878	1	7.2454	36	42.6438
2	56.0686	37	62.3277	2	44.9220	37	42.6536	2	76.7831	37	88.2800	2	45.9518	37	59.9886	2	42.4460	37	57.5284
3	51.0850	38	53.8149	3	29.8205	38	45.6266	3	88.2800	38	74.1726	3	57.3066	38	49.3658	3	35.4969	38	62.3191
4	30.3950	39	54.6463	4	10.2679	39	82.4349	4	3.1957	39	88.2800	4	8.6656	39	74.2058	4	21.1137	39	62.3722
5	37.4872	40	57.4868	5	30.3284	40	62.4833	5	36.4190	40	74.3477	5	7.0983	40	54.4365	5	34.0488	40	46.6877
6	24.6963	41	3.0000	6	39.1328	41	3.5698	6	41.6098	41	5.4863	6	35.8760	41	5.4696	6	55.6339	41	3.8544
7	44.4298	42	3.0000	7	62.0522	42	33.2252	7	48.4907	42	3.6124	7	46.7428	42	3.1025	7	35.2796	42	28.1173
8	25.5762	43	3.0000	8	39.0820	43	5.4702	8	10.1905	43	4.4459	8	44.3974	43	3.0778	8	21.0913	43	4.3737
9	51.6692	44	33.3543	9	21.2109	44	37.4164	9	34.4618	44	63.0352	9	4.3259	44	3.4710	9	23.8671	44	45.4013

10	26.0943	45	47.4641	10	47.7581	45	42.9331	10	77.7641	45	82.7444	10	41.3986	45	68.9757	10	41.1491	45	40.6756
11	46.1273	46	41.0117	11	43.5721	46	43.6959	11	82.8627	46	3.0000	11	45.9778	46	47.1158	11	49.4592	46	38.4261
12	39.0085	47	38.9506	12	50.1162	47	40.0182	12	4.9180	47	31.9705	12	41.7316	47	9.1771	12	40.2268	47	40.6960
13	42.2062	48	48.4108	13	15.5862	48	6.1057	13	45.7692	48	56.0556	13	5.6217	48	48.0278	13	20.2190	48	44.0171
14	45.3889	49	44.2000	14	34.1207	49	69.4682	14	18.7481	49	8.5576	14	43.8835	49	53.0359	14	43.9537	49	47.8302
15	21.3996	50	23.1173	15	41.3569	50	16.1663	15	80.1337	50	34.9689	15	40.5803	50	6.1905	15	41.4136	50	3.1699
16	55.8747	51	25.2248	16	34.0594	51	52.8570	16	3.6303	51	6.0126	16	43.2366	51	3.9220	16	26.3408	51	25.5141
17	54.6989	52	3.0000	17	35.8065	52	4.1141	17	26.0943	52	16.5495	17	5.9044	52	46.7173	17	13.8420	52	42.8114
18	44.3630	53	3.0000	18	34.4436	53	3.4906	18	88.2800	53	44.3998	18	41.5151	53	9.9630	18	45.1544	53	3.1115
19	69.2845	54	45.0476	19	42.0667	54	10.7157	19	36.5706	54	3.2490	19	44.1385	54	3.0993	19	43.4024	54	3.2235
20	36.3042	55	33.2372	20	41.1269	55	42.6278	20	3.6994	55	7.2914	20	42.6853	55	44.1764	20	34.6751	55	40.3892
21	51.4064	56	51.3823	21	29.8335	56	20.0221	21	22.8482	56	50.2551	21	7.4434	56	7.8372	21	20.2150	56	23.4285
22	59.0334	57	34.0133	22	42.4877	57	48.5791	22	56.4145	57	88.2800	22	43.4209	57	50.2297	22	37.5687	57	44.3394
23	52.5156	58	3.0000	23	53.2635	58	21.4132	23	72.4224	58	3.0000	23	55.4360	58	44.1736	23	43.3777	58	35.4990
24	88.2800	59	52.5090	24	35.1364	59	38.1539	24	8.0787	59	12.4085	24	39.9090	59	3.2029	24	48.6701	59	35.8941
25	27.8664	60	29.8101	25	26.1959	60	15.6696	25	33.8972	60	68.8886	25	3.0193	60	45.1477	25	33.4961	60	32.2447
26	60.8462	61	37.8282	26	44.6208	61	50.2004	26	62.3369	61	5.0984	26	21.8692	61	44.8072	26	35.5172	61	34.4426
27	49.4879	62	35.9664	27	33.7693	62	7.9297	27	52.1827	62	50.7306	27	45.1819	62	3.0598	27	16.6273	62	45.2556
28	35.6458	63	3.0000	28	49.6919	63	43.6860	28	4.8242	63	10.9981	28	44.3409	63	7.8814	28	37.8035	63	3.9604
29	35.2384	64	3.0000	29	8.3112	64	3.6401	29	27.3140	64	3.0000	29	3.0256	64	42.1576	29	40.4308	64	4.5185
30	50.7653	65	3.0000	30	45.7488	65	5.8682	30	28.0415	65	3.0000	30	30.4577	65	3.4860	30	35.1539	65	3.6150
31	26.9287	66	3.0000	31	24.8783	66	9.8094	31	31.4979	66	47.0715	31	23.9756	66	3.2132	31	12.3125	66	3.1877
32	25.1350	67	4.7347	32	42.9816	67	3.4524	32	4.4834	67	3.0000	32	43.6214	67	18.5162	32	46.3373	67	5.5657
33	25.7039	68	39.4951	33	12.2387	68	4.1140	33	22.1488	68	40.7262	33	8.8847	68	44.1659	33	35.1358	68	48.4156
34	30.9292	69	3.0000	34	47.3799	69	43.2039	34	49.5387	69	62.1344	34	34.4806	69	5.5788	34	45.3763	69	5.7537
35	47.1990	70	69.5588	35	26.4803	70	45.6091	35	49.5348	70	5.0830	35	20.9954	70	48.3381	35	4.6655	70	44.2022

Table 6: Optimization results obtained for the planar ten-story frame

Statistical index	PSO [29]	CSS [29]	GWO [29]	IGWO [29]	ACSS (present study)
Best weight (lb)	140127.5309	130115.2747	143448.7343	120635.7922	128370.0454
Average weight (lb)	158293.3047	133489.9963	153209.0455	125677.0802	133027.6469
Standard deviation weight (lb)	7532.3112	2027.1734	4204.8825	2980.1319	2213.4142

Table 7: Natural frequencies (Hz) computed for optimized designs of the planar ten-story frame

Mode no.	PSO [29]	CSS [29]	GWO [29]	IGWO [29]	ACSS (present study)
1	2.0000	2.0060	2.2755	2.0408	2.0029
2	7.0000	7.0095	7.0971	7.0000	7.0020
3	15.0000	15.0006	15.1538	15.0000	15.0003

6. CONCLUSIONS

This study uses a recently developed metaheuristic algorithm known as the adaptive charged system search (ACSS) for optimal design of planar frame structures under natural frequency constraints. The ACSS is an enhanced version of the CSS, which contains an improved initialization procedure as well as an improved updating procedure.

Three design examples including two-story, seven-story and ten-story planar frames are considered for the optimization. Results demonstrate the desirable performance of the ACSS, as indicated by the statistical indices like the average, standard deviation and convergence history. The ACSS outperforms the CSS and provides suitable stability according to the standard deviation of the solutions. However, the improved grey wolf optimizer (IGWO) provides better solutions than the ACSS in the test cases investigated here. This study also shows that the ACSS can provide desirable solutions for optimal design of moment-resisting frames subjected to frequency constraints.

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