



OPTIMUM COST DESIGN OF REINFORCED CONCRETE SLABS USING A METAHEURISTIC ALGORITHM

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ABSTRACT

In this article, the optimum design of a reinforced concrete solid slab is presented via an efficient hybrid metaheuristic algorithm that is recently developed. This algorithm utilizes the mouth-brooding fish (MBF) algorithm as the main engine and uses the favorable properties of the colliding bodies optimization (CBO) algorithm. The efficiency of this algorithm is compared with mouth-brooding fish (MBF), Neural Dynamic (ND), Cuckoo Search Optimization (COA) and Particle Swarm Optimization (PSO). The cost of the solid slab is considered to be the objective function, and the design is based on the ACI code. The numerical results indicate that this hybrid metaheuristic algorithm can to construct very promising results and has merits in solving challenging optimization problems.

Keywords: mouth brooding fish; cost optimization; meta-heuristic algorithms; colliding bodies optimization; solid slab; reinforced concrete.

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1. INTRODUCTION

Optimization algorithms can be divided into two general categories of gradient-based methods and metaheuristics. Population-based meta-heuristic algorithms consists of two phases: an exploration of the search space and exploitation of the best solutions found. One of the most important subjects in a good metaheuristic algorithm is to keep a reasonable balance between the exploration and exploitation abilities [1].

Meta-heuristic optimization algorithms are becoming more and more popular in engineering applications because they: (i) rely on rather simple concepts and are easy to implement; (ii) do not require gradient information; (iii) can bypass local optima; (iv) can be utilized in a wide range of problems covering different disciplines [2].

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Nature-inspired meta-heuristic algorithms can be grouped in three main categories: evolution-based, physics-based, and swarm-based methods. Evolution-based methods are inspired by the laws of natural evolution. The most popular evolution-inspired techniques are Genetic Algorithms (GA) that simulates the Darwinian evolution, Probability-Based Incremental Learning (PBIL), Genetic Programming (GP), and Biogeography-Based Optimizer (BBO).

Physics-based methods imitate the physical rules in the universe. The most popular algorithms are Simulated Annealing (SA), Gravitational Local Search (GLSA), Big-Bang Big-Crunch (BBBC), Gravitational Search Algorithm (GSA), Charged System Search (CSS) [3], Central Force Optimization (CFO), Artificial Chemical Reaction Optimization Algorithm (ACROA), Black Hole (BH) algorithm, Ray Optimization (RO) algorithm, Small-World Optimization Algorithm (SWOA), Galaxy-based Search Algorithm (GbSA), Curved Space Optimization (CSO), Colliding Bodies Optimization (CBO) [4,5], water evaporation optimization (WEO) and Big Bang–Big Crunch algorithm (BB–BC).

The third group of nature-inspired methods includes swarm-based techniques that mimic the social behavior of groups of animals. The most popular algorithm is Particle Swarm Optimization (PSO) [6], Ant Colony Optimization (ACO), Marriage in Honey Bees Optimization Algorithm (MBO), Artificial Fish-Swarm Algorithm (AFSA), Termite Algorithm, ABC, Wasp Swarm Algorithm, Monkey Search, Wolf pack search algorithm, Bee Collecting Pollen Algorithm (BCPA), Cuckoo Optimization Algorithm (COA), Dolphin Partner Optimization (DPO), Bat-inspired Algorithm (BA), Firefly Algorithm (FA), Hunting Search (HS), Bird Mating Optimizer (BMO), Krill Herd (KH), Fruit fly Optimization Algorithm (FOA), Dolphin Echolocation (DE) and Mouth Brooding Fish algorithm (MBF) [7].

It is worth mentioning here that there are also other meta-heuristic methods inspired by human behaviors in the literature. Some of the most popular algorithms are Teaching Learning Based Optimization (TLBO), Harmony Search (HS) [8], Tabu (Taboo) Search (TS), Group Search Optimizer (GSO), Imperialist Competitive Algorithm (ICA), League Championship Algorithm (LCA), Firework Algorithm, Interior Search Algorithm (ISA), Mine Blast Algorithm (MBA), Soccer League Competition (SLC) algorithm, Seeker Optimization Algorithm (SOA), Social-Based Algorithm (SBA), Exchange Market Algorithm (EMA), and Group Counseling Optimization (GCO) algorithm.

One of the recently developed metaheuristics is MBF-CBO [9]. This algorithm utilizes the mouth-brooding fish algorithm as the main engine and uses the favorable properties of the colliding bodies optimization algorithm to find the best possible answer.

The main objective of the present study is to minimize one objective function under some specific limitations. Thus, in this paper, this hybrid metaheuristic algorithm is used for the optimum design of a reinforced concrete solid slab. The results of design are also compared with previous literature for example Application of probabilistic particle swarm in optimal design of large-span prestressed concrete slabs [10], Cost optimization of a composite floor system, one-way waffle slab, and concrete slab formwork using Charged System Search algorithm [11], Harmony search based algorithm for the optimum cost design of reinforced concrete one-way ribbed slabs [12] and Harmony search algorithm for optimum design of slab formwork [13].

The present paper is organized as follows: In the next section, standard algorithm is briefly introduced. Section 3 consisting of the study of optimization of one civil constrained function. Conclusion is presented in section 4.

2. HYBRID OPTIMIZATION ALGORITHM (MBF-CBO)

2.1 Mouth-brooding fish algorithm (MBF)

The mouth-brooding fish algorithm (MBF) by Jahani and Chizari (2018) is a popular metaheuristic algorithm that is based on the life-cycle of mouth-brooding fish. The MBF algorithm uses the movements of the mouth-brooding fish and their children's struggle for survival as a pattern to find the best possible answer. This algorithm has five controlling parameters that the user determines. These parameters are the number of population of cichlids (nFish), the mother's source point (SP), the amount of dispersion (Dis), the probability of dispersion (Pdis), and the mother's source point damping (SPdamp). The most important parameter of an MBF algorithm is how the cichlids surround their mother or, in other words, move around her and the impacts of nature on their movements. This algorithm has used for designing of slabs for example the optimum design of a reinforced concrete one-way ribbed slab [14].

2.2 Colliding bodies optimization algorithm (CBO)

The colliding bodies optimization (CBO) by Kaveh and Mahdavi (2014) is a population-based meta-heuristic algorithm inspired by a one-dimensional collision between bodies. This algorithm starts with a random population of colliding bodies. The masses of these bodies are calculated according to their objective function values. The agents are sorted in ascending order of their fitness values and then divided into two equal groups, i.e., stationary and moving groups. The lower half of the agents are stationary groups, and the rest of them are moving. The moving agents move towards the stationary agents, and a collision happens between the pairs of agents. This collision motivates the agents to move toward better positions in the search space. The repetition of these actions leads to the reaching of an optimal position in the search space, or after a predefined maximum evaluation number, the optimization process is terminated. This algorithm has had many applications in civil engineering for example Optimization of Haraz dam reservoir operation using CBO metaheuristic algorithm [15] and Optimum cost design of reinforced concrete one-way ribbed slabs using CBO, PSO and Democratic PSO algorithms [16].

2.3 MBF-CBO based hybrid optimization algorithm

In order to modify and improve the updating mechanism of the standard MBF, a modified version of the MBF is considered. This proposed modification is inspired by the CBO algorithm because this algorithm does not utilize any internal parameters. This study attempts to enhance the original formulation of the MBF by hybridizing it with some concepts of the colliding bodies optimization (CBO) in order to improve solution accuracy, reliability and convergence speed.

In this modified version of the MBF [9], all the cichlids are sorted according to their objective function values in ascending order. The cichlids are divided into two equal groups. The lower half of the cichlids are called "explorer" cichlids; these cichlids are excellent agents, while the upper half of the cichlids are called "imitator" cichlids. Unlike the standard MBF, not all the search agents update their positions towards only the best search agent. Instead, the explorer cichlids update their positions towards the upper half of the cichlids to find better solutions, and the imitator cichlids update their position towards the lower half of the cichlids to improve their position.

Another modification is considering a memory to save some best agents in each iteration.

The steps of the proposed algorithm are as follows:

Step 1: Initialization

The initial positions of all the cichlids are determined randomly in the search space.

Step 2: Evaluation in terms of the fitness of the cichlids

The fitness value of each cichlid is calculated according to the objective function of the optimization problem.

Step 3: Arrangement of populations

All the cichlids are sorted according to their fitness values in ascending order. Then the cichlids are divided into two equal groups.

Step 4: Saving

The considered memory is updated in each iteration according to the calculated fitnesses. Afterwards, the members of the updated memory are added to the population, and the same number of the worst cichlids are deleted.

Step 5: Updating the cichlid's position

For updating each cichlid's position in the main movements (modified), A_{sp} , A_{lb} , A_{gb} , A_{nf} , and A_{ex} are calculated by Eqs. (1), (3), (4), (6), and (7), respectively.

$$A_{sp} = SP \times \text{Cichlids} \cdot \text{Movements} \quad (1)$$

where SP is the mother's source point and cichlids, the movements are the last movements of the cichlids.

$$SP = SP \times \text{SPdamp} \quad (2)$$

where SP is the mother's source point that changes for the next iteration, and SPdamp is the mother's source point damp and varies between 0.85 and 0.95.

$$A_{lb} = \text{Dis} \times (\overrightarrow{X_{lb}} - \overrightarrow{X_l}) \quad (3)$$

$$A_{gb} = \text{Dis} \times (\overrightarrow{X_{gb}} - \overrightarrow{X_l}) \quad (4)$$

where $\overrightarrow{X_l}$, $\overrightarrow{X_{lb}}$, and $\overrightarrow{X_{gb}}$ are the previous positions, the local best and global best of the cichlids, respectively. Dis is the amount of dispersion that is one of the controlling parameters that is selected by the user and could increase or decrease the effect of this movement.

$$\text{NewN} \cdot \text{F} \cdot \text{P} = 10 \times \text{SP} \times \text{NatureForce} \cdot \text{Position} \quad (5)$$

where NatureForce.Position(SelectedCells) is the selected cell from 60 percent of the different cells of the best position of the last and current generation.

$$A_{nf} = \text{Dis} \times (\text{NewN} \cdot \text{F} \cdot \text{P} - \text{NatureForce} \cdot \text{Position}) \quad (6)$$

where natureForce.position is the best position of the cichlids of the last iteration.

$$A_{ex} = \text{Dis} \times (\overrightarrow{X_{l-n/2}} - \overrightarrow{X_l}) \quad (7)$$

The new positions of the cichlids are evaluated using the following expressions:

$$\overrightarrow{X}_i^{new} = \overrightarrow{X}_{best} + (\overrightarrow{A}_{sp} + \overrightarrow{A}_{lb} + \overrightarrow{A}_{gb} + \overrightarrow{A}_{nf}) \quad i=1,2,\dots,n/2 \quad (8)$$

$$\overrightarrow{X}_i^{new} = \overrightarrow{X}_{i-n/2} + (\overrightarrow{A}_{sp} + \overrightarrow{A}_{ex} + \overrightarrow{A}_{nf}) \quad i=n/2+1, n/2+2, \dots, n \quad (9)$$

where $\overrightarrow{X}_i^{new}$ and \overrightarrow{X}_i are a new position and a previous position of the cichlids, respectively.

The effects of the nature (A_{nf}) trends only occur when the whole convergence trend is not in good shape.

If the best cost of the current iteration was better than the best cost of the last iteration (at most 15 percent), the new positions of the cichlids are evaluated using the following expression:

$$\overrightarrow{X}_i^{new} = \overrightarrow{X}_{best} + \overrightarrow{UASDP} \quad i=1,2,\dots,n/2 \quad (10)$$

$$\overrightarrow{X}_i^{new} = \overrightarrow{X}_{i-n/2} + \overrightarrow{UASDP} \quad i=n/2+1, n/2+2, \dots, n \quad (11)$$

where UASDP and UASDN are calculated by Eqs. (12) and (13).

$$ASDP = 0.1 \times (\text{VarMax} - \text{VarMin}) \quad ASDN = -ASDP \quad (12)$$

$$UASDP = 4 \times ASDP \quad UASDN = -UASDP \quad (13)$$

where VarMin and VarMax are the minimum and maximum limits of the variations problems, respectively, and finally, updating each cichlid's position on the additional movements, Crossover, and Shark attack without any change.

Step 6: Check of condition of termination

The optimization is repeated from Step 2 until a termination criterion as the maximum number of iterations is satisfied.

3. NUMERICAL EXAMPLE: COST OPTIMIZATION OF REINFORCED CONCRETE SOLID SLABS

The performance of the MBF-CBO algorithm is studied through one example of RC solid slabs with simply support at both ends taken from the optimization literature. This example is independently optimized 30 times, and the MBF algorithm ran 1000 iterations.

3.1. Models formulation

In a reinforced concrete one-way slab optimization problem, the aim is to minimize the cost of the structure while satisfying some constraints. The three discrete design variables selected for modelling of the slab are shown in Fig. 1. These include the thickness of the slab (h), the bar diameter (d_b) and the spacing of the reinforcement bars (s).

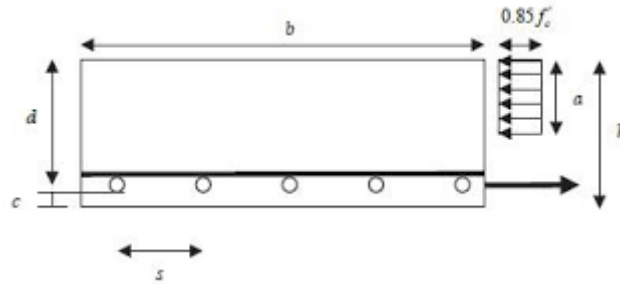


Figure 1. Schematic view of a reinforced concrete slab

3.2 Optimum design process

Typical design of the slabs consists of two steps:

- Selecting random values for the variables and controlling the dimensions.
- Calculating the required reinforcement and controlling the strength according to the ACI.

3.3 Objective function

The objective function of solid slab includes the costs associated with concrete and steel material as well as concreting and erecting the reinforcement. The minimum of these costs determines the optimal design of the concrete slab. This can be attained by determining the optimal values for decision variables h , d_b and s . The objective function can be expressed as follows:

$$Q = (V_{\text{conc}} \times C_c) + (W_{\text{steel}} \times C_r) \quad (14)$$

where V_{conc} and W_{steel} are the volumes of concrete and the weight of the reinforcement steel in the unit length (m³, kg), respectively; C_c and C_r are the costs of concrete and steel (\$/m³ for concrete and \$/kg for steel), respectively. (The formwork and finishing cost does not vary significantly for any given locality and consequently dropped from the formulation)

$$V_{\text{conc}} = L * b * h \quad (15)$$

$$W_{\text{steel}} = W_s * L * A_s \quad (16)$$

where L , b , h , W_s and A_s are the span length, the span width, the thickness of slab (Fig. 1), unit weight (specific weight per unit volume) of steel and cross-section area of reinforcement bars, respectively. The quantity A_s is calculated by

$$A_s = \pi d_b^2 / 4 (b/s) \quad (17)$$

where d_b and s are the diameter and the spacing of the reinforcement bars, respectively.

3.4 Design constraints

The formulation of the optimal design problem is carried out according to the provisions provided in Ahmadkhanlou and Adeli [17].

3.4.1 Flexural Constraint

The flexural constraint can be expressed as follows:

$$M_u/(\phi_b M_n) \leq 1 \quad \phi_b = 0.9 \quad (18)$$

where M_n and M_u are the nominal bending moment and the ultimate design moment, respectively.

$$M_u = kwL_n^2 \quad (19)$$

where l_n and k are, respectively, the clear span length and the moment coefficient for a continuous slab. The values of k are provided in Table 1. In Eq. (19), the maximum value of moment coefficient for four different support conditions (simply-supported, continuous at one end and simply-supported at the other, continuous at both ends, and cantilever) is used which is given in Table 2.

In Eq. (19), w is the factored uniformly distributed load.

$$w = 1.4(DL * b + DLs) + 1.7(LL * b * h) \quad (20)$$

in which DL , LL , and DLs are the dead load of floor excluding the self-weight of the slab, live load, and self-weight of the slab. DLs is calculated as follows:

$$DLs = (b * h - A_s)W_c + A_s * W_s \quad (21)$$

where W_c is the weight of the concrete per unit volume.

Table 1. Moment coefficient for continuous slabs

| Exterior span | | Interior span | | | |
|---------------|--------|---------------|--------|---------|--------|
| Support | Middle | Support | Middle | Support | Middle |
| -1/24 | +1/14 | -1/10 | -1/11 | +1/16 | -1/11 |

Table 2. Maximum Moment coefficient, k , used for the design of RC slabs

| Simply Supported | One end continuous | Both ends continuous | Cantilever |
|------------------|--------------------|----------------------|------------|
| 1/8 | 1/10 | 1/11 | 1/2 |

The nominal bending moment, M_n , is calculated as follows:

$$M_n = A_s * f_y (d - a/2) \quad (22)$$

$$a = (A_s * f_y) / (0.85 * f_c * b) \quad (23)$$

where f_c is the specified compressive strength of concrete and d is (h-c).

3.4.2 Shear constraint

The shear constraint is presented in the following form:

$$V_u / (\phi_v V_n) \leq 1 \quad (24)$$

where V_u and V_n are the ultimate factored shear force and the nominal shear strength of the concrete, respectively.

The ultimate factored shear force is defined as follows:

$$V_u = k_v * w * Ln/2 \quad (25)$$

where k_v is the shear coefficient for a continuous slab that depends on the type of slab supports. The values of k_v are given in Table 3. The nominal shear strength of concrete is defined as follows:

$$V_n = 2\sqrt{f_c} * b * d \quad (26)$$

Table 3: Shear coefficient for continuous slabs

| Simply Supported | One end continuous | Both ends continuous | Cantilever |
|------------------|--------------------|----------------------|------------|
| 1 | 1.15 | 1 | 2 |

3.4.3 Serviceability Constraints

The serviceability constraints are presented in terms of the limits on the steel reinforcement ratio and the bar spacing(s). The steel reinforcement ratio should satisfy the following constraint:

$$\rho \leq \rho_{max} = 0.75\rho_b \quad (27)$$

$$\rho_b = 0.85\beta_1 * f_c / f_y * \left(\frac{87000}{87000 + f_y} \right) \quad (28)$$

and β_1 is calculated from

$$\text{For } f_c \leq 4000 \text{ psi } \beta_1 = 0.85 \quad (29)$$

$$\text{For } f_c > 4000 \text{ psi } \beta_1 = 0.85 - 0.05 \left(\frac{f_c - 4000}{1000} \right) \geq 0.65 \quad (30)$$

The minimum shrinkage steel ratio, ρ_{min} , in the slab is 0.002 for slabs in which bars of grade 40 or 50 are utilized and 0.0018 for slabs in which deformed bars of grade 60 are used. The bar spacing should satisfy the following constraints:

- The minimum clear spacing between bars in a layer, d_b , should not be less than 1 in.
- The maximum spacing between the bars ≤ 3 times the rib thickness ≤ 18 in. (450 mm).

3.4.4 Deflection Constraints

Based on the ACI code h_{min} of L/20, L/24, L/28, or L/10 is required, depending on the support

conditions (Simply Supported, one end continuous, both ends continuous, Cantilever, respectively), with an absolute minimum thickness of 1.5 in (38.1 mm). The values of h_{\min} are applicable for normal weight concrete and $f_y = 60,000$ psi. For f_y other than 60,000 psi, the values shall be multiplied by α_1 (which is given in Eq. (31)). For lightweight concrete having w_c in the range of 90 to 115 lb/ft³, the values shall be multiplied by α_2 (which is given in Eq. (32)).

$$\alpha_1 = 0.4 + (f_y/10000) \quad (31)$$

$$\alpha_2 = \max(1.65 - 0.005W_c, 1.09) \quad (32)$$

3.5 Design

One-way RC slabs were previously optimized with MBF by Davood Sedaghat Shayegan, Alireza Lork and Seyed Amir Hossein Hashemi [18], neural dynamics (ND) model by Ahmadvkhanlou and Adeli, PSO by Varae and Ahmadi-Nedushan [19] and COA by Ghandi, Shokrollahi and Nasrolahi [20]. In this paper one-way RC slab (with simply supported at both ends) is optimized with MBF-CBO. The results of the examples are compared to PSO, neural dynamics model and MBF.

The general data for the example is provided in Table 4. The results of the optimal design are provided in Table 5 and Convergence curve of the MBF-CBO algorithm is shown in Fig. 2.

Table 5 compares the results of the optimal design for the one-way reinforced concrete slab attained by the Neural dynamic (ND), cuckoo search optimization (COA), Particle Swarm Optimization (PSO), Mouth-Brooding Fish (MBF) and MBF-CBO (present study) algorithms.

Investigation of the convergence curve in Fig. 2 shows that downfall of the curve, in initial steps, demonstrates the power of the method in exploration. Then, a local search is started and, in 37 iterations, the minimum solution is found (this value for COA is about 11). According to Table 5 and Fig. 2, the MBF-CBO algorithm has acceptable performance and speed of convergence to optimize the RC slab.

Table 4. General data

| | |
|--------|------------------------|
| f_y | 40 ksi |
| f'_c | 3 ksi |
| DL | 10 lb/ft ² |
| LL | 40 lb/ft ² |
| L | 13 ft |
| Cover | 0.75 in |
| Ws | 490 lb/ft ³ |
| Wc | 150 lb/ft ³ |
| b | 1 ft |
| Cc | 76 \$/cyd |
| Cr | 1300 \$/ton |

Table 5. Results of the optimization

| Algorithm | ND [17] | PSO [19] | MBF [18] | COA [20] | Present study (MBF-CBO) |
|---------------------|------------|-------------|-------------|-------------|----------------------------|
| Slab thickness (in) | 6.75 | 6.25 | 6.24 | 6.25 | 6.04 |

| | | | | | |
|-------------------|-------|-------|-------|-------|-------|
| Bar diameter (in) | 0.375 | 0.5 | 0.439 | 0.375 | 0.385 |
| bar spacing (in) | 6.5 | 9 | 8.12 | 14.5 | 8.17 |
| Total cost (\$) | 26.45 | 26.57 | 26.34 | 26.36 | 26.15 |

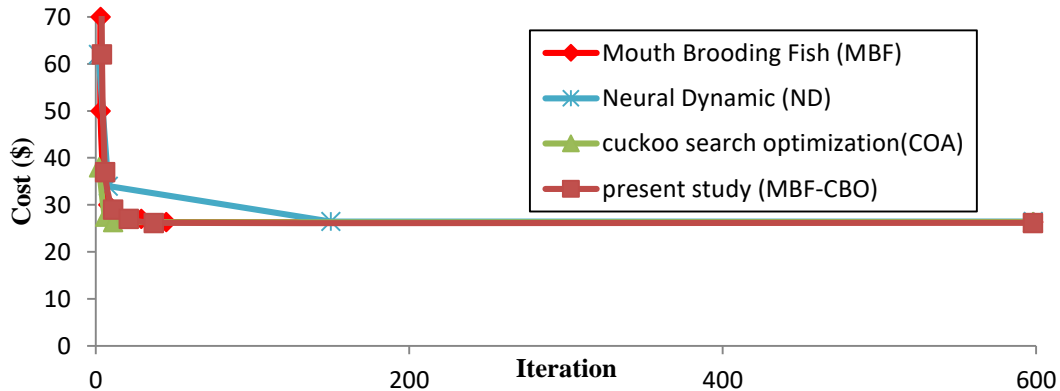


Figure 2. Convergence curves of the MBF-CBO, MBF, COA and ND algorithms for RC Slabs

4. CONCLUSIONS

This study uses the MBF-CBO algorithm for solving optimization problems and in particular for structural cost optimization of a solid slab. The design of the slabs was based on ACI code, and the procedure included finding the optimum thickness of the slab, the diameter of reinforcement bars, and spacing of reinforcement. The constraints were handled using the penalty function. The main objective of this article is to study the convergence curve of this method for a solid slab and compare the obtained values with results of mouth-brooding fish (MBF), Neural Dynamics (ND) and Particle Swarm Optimization (PSO).

The results obtained show that the MBF-CBO method is powerful and efficient approaches for finding the optimum solution to structural optimization problems. The power comes from the fact that is, the downfall of the curve, in initial steps as demonstrated in Fig. 2. Furthermore, for the solid slabs, the comparison of the optimization results of MBF-CBO with MBF, neural dynamics model and PSO shown the superiority of the MBF-CBO to achieve better results than the other three algorithms.

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