SEISMIC RELIABILITY ASSESSMENT OF OPTIMALLY DESIGNED STEEL CONCENTIRCALLY BRACED FRAMES

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ABSTRACT

The main aim of this study, is to evaluate the seismic reliability of steel concentrically braced frame (SCBF) structures optimally designed in the context of performance-based design. The Monte Carlo simulation (MCS) method and neural network (NN) techniques were utilized to conduct the reliability analysis of the optimally designed SCBFs. Multi-layer perceptron (MLP) trained by back propagation technique was used to evaluate the required structural responses and then the total exceedence probability associated with the seismic performance levels was estimated by the MCS method. Three numerical examples of 5-, 10-, and 15-story SCBFs with fixed and optimal topology of braces are presented and their probability of failure was evaluated considering the resistance characteristics and the seismic loading of the structures. The numerical results indicate that the SCBFs with optimal topology of braces were more reliable than those with fixed topology of braces.

1. INTRODUCTION

One of the main concerns in structural engineering is the design of cost-efficient structures with acceptable performance against earthquakes. On the other hand, performance-based design (PBD) [1] is a modern seismic design procedures for the rehabilitation of existing structures and the seismic design of new ones. So, structural optimization methodologies have been developed in the last decades and structural performance-based design optimization (PBDO) has become a topic of growing interest [2-10] in the field of structural

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engineering. As can be seen from literature, metaheuristics are the best choice to deal with the PBDO problems, because they are powerful algorithms for exploring and exploiting the design space and are also simple for computer implementation.

The intrinsic random nature of material properties and actions must be actually considered in the design process of structures and the probability of failure must be computed from the joint probability distribution of the random variables associated with the action and resistance. Theory and methods for structural reliability are actually useful tools for evaluating the safety of complex structures. Recent developments allow anticipating that their application will gradually increase, even in the case of common structures [11]. Monte Carlo Simulation (MCS) is a simulation method for reliability analysis. The main concept of simulation techniques is to simulate a probabilistic phenomenon numerically and then observe the frequency of a certain event in that phenomenon [12]. These simulation techniques are easy to implement, but in the case of small failure probabilities, the number of simulations required is extremely large which greatly increases the computational cost of these simulation techniques. Thus, the MCS method can be applied to many practical problems that allows direct consideration of any type of probability distribution for random variables. This method is able to calculate the probability of failure with the desired precision. However, its computational burden is high because the MCS requires a large number of structural analyses [13]. In the current work, the reliability theory and PBD approach were simultaneously utilized to evaluate the reliability index of optimally designed steel concentrically braced frame (SCBF) structures for earthquake loadings. In order to address the uncertainties in material properties and seismic actions, structural nonlinear responses were required to perform reliability analysis using the MCS method. As a result, the computational cost required for this process will be expensive. One of the best candidates for reducing the computational burden of the reliability analysis is neural network (NN) techniques. In this study, feed-forward multi-layer perceptron (MLP) trained by back propagation technique [14] in MATLAB [15] platform was used to evaluate the required structural responses.

Reliability analysis of the 5-, 10-, and 15-story SCBFs with fixed and optimal topology of braces designed in the framework of PBD was conducted in the present study. For each optimally designed structure a NN model is trained to provide the data required to perform the reliability analysis using the MCS method. The obtained numerical results demonstrate that the reliability index of the SCBFs with optimal topology of braces were higher compared to the SCBFs with fixed topology of braces.

2. OPTIMAL SEISMIC DESIGN OF SCBF STRUCTURES

In the present work, immediate occupancy (IO), life safety (LS) and collapse prevention (CP) are considered as the performance levels and three hazard levels with 50%, 10% and 2% probability of exceedance in 50 year period (50%/50y, 10%/50y, and 2%/50y) are adopted according to the hazard model of Standard No. 2800 [16] as shown in Fig. 1. The nonlinear static pushover analysis is performed to quantify seismic induced nonlinear response of structures according to [1]. In this method the structure is pushed with a specific distribution of the lateral loads until the target displacement is reached. *OpenSees* [17] is

utilized to conduct the pushover analysis. In addition, to capture the buckling behavior of braces, uniaxial phenomenological model [18] was used.
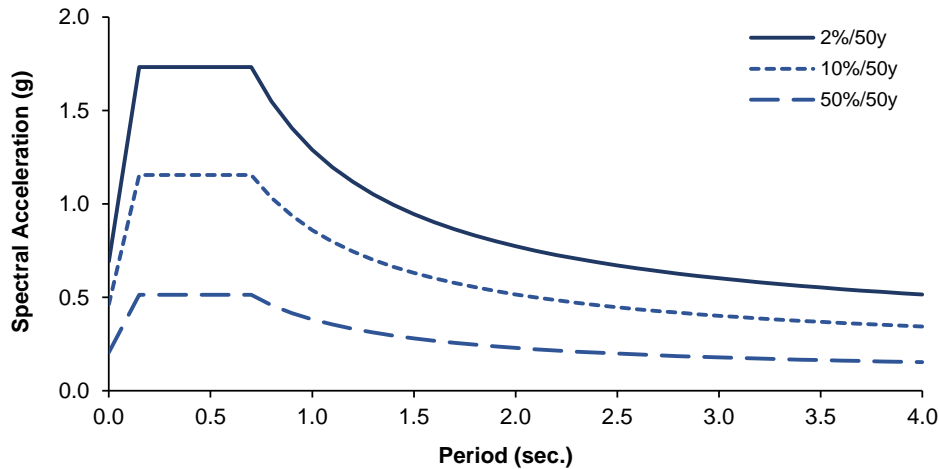

Figure 1. Acceleration response spectra of hazard levels

The aim of the PBDO process is to minimize the weight of the structure under some constraints. For a SCBF consisting of *ne* members that are collected in *ng* design groups, the discrete optimization problem can be formulated as follows:

$$\text{Minimize: } w(X) = \sum_{i=1}^{ng} \rho_i A_i \sum_{j=1}^{nm} L_j \tag{1}$$

$$\text{Subject to: } g_k(X) \leq 0, \; k = 1, 2, \cdots, nc \tag{2}$$

where X is vector of design variables including cross-section of elements and placement of braces in the frame; w represents the weight of the frame, ρ_i and A_i are weight of unit volume and cross-sectional area of the ith group section, respectively; nm is the number of elements collected in the ith group; L_j is the length of the jth element in the ith group; $g_k(X)$ is the kth behavioral constraint.

Three types of constraints including geometric, strenght and PBD constraints were checked during the optimization process. The geometric constraints must be satisfied in framing joints to meet the practical demands of construction. The strength constraints of structural elements were checked for gravity loads to perform serviceability checks based on AISC-LRFD [19] design code. The PBD constraints including inter-story drift, plastic rotation of columns and plastic deformation of braces were cheched according to FEMA-356 [1] and ASCE 41-13 [20] to ensure the desired seismic performance of the structures.

Over the recent years, many efficient metaheuristic algorithms have been proposed to deal with the complex structural optimization problems such as colliding bodies optimization (CBO) [21], enhanced colliding bodies optimization (ECBO) [22], and center of mass optimization (CMO) [10]. The CMO was proposed based on the concept of center of mass in physics. This metaheuristic algorithm is an efficient and powerfull tool to tackle the PBDO problems of structures.

3. RELIABILITY ANALYSIS

Obviously, modelling uncertainty plays an important role in evaluating the seismic reliability of structures. There are numerous sources of uncertainties which may seriously affect the structural seismic performance. Among these sources, material properties and seismic loads are involved as the uncertain variables in the current work. The structural nonlinear analysis should be performed to evaluate the probabilistic structural response. Then, limit state functions associated with each performance level should be calculated using the probabilistic structural response. As a result, the non-performance probability related to each performance level can be evaluated using the MCS method. Overall, the MCS is a simple and powerful tool for solving a wide range of reliability problems. However, using it to assess very low probabilities of failure requires a large number of structural analyses to be conducted that can be excessively time consuming. In order to address this critical issue, an efficient NN model in conjunction with the MCS is used in this study to significantly reduce the computational cost of seismic reliability assessment of structures. the employed NN model is feed-forward multi-layer perceptron (FFMLP) trained by back propagation technique using neural network toolbox of MATLAB. The following subsections briefly describe the mathematical background of MCS and FFMLP.

3.1 Monte Carlo simulation
To solve reliability problems, random variables must be defined. For the SCBFs optimally designed in the framework of PBD, the random variables are considered as follows:

$$U = \left\{ E \quad f_y \quad S_a^{IO} \quad S_a^{LS} \quad S_a^{CP} \right\}^{\mathrm{T}} \tag{3}$$

where U is vector of random variables; E and f_y are respectively Young's modulus and yield strength of steel materials considered for the uniaxial phenomenological model; S_a^{IO}; S_a^{LS} and S_a^{CP} are spectral acceleration of the hazard levels of the optimally designed SCBFs.

A reliability problem is normally formulated using a limit state function. Limit state function for each performance level is defined using capacity and demand as follows:

$$G^i(U) = R_L^i - R^i(U) \,,\, i = IO; LS; CP \tag{4}$$

where G is a limit state function; R_L is the limiting value for a seismic response $R(Z)$.

In the reliability analysis performed in this work, the maximum inter-story drift ratios at the IO, LS and CP performance levels are selected as the structural seismic responses, and subsequently, the considered limit state functions for the performance levels are as follows:

$$G^{IO}(U) = 0.005 - \delta_{max}^{IO}(U) \tag{5}$$

$$G^{LS}(U) = 0.015 - \delta_{max}^{LS}(U) \tag{6}$$

$$G^{CP}(U) = 0.020 - \delta_{max}^{CP}(U) \tag{7}$$

where the limiting values for the maximum inter-story drift ratios at the IO, LS and CP performance levels are taken as 0.005, 0.015, and 0.020 [1], respectively; δ_{max}^{IO}, δ_{max}^{LS}, and δ_{max}^{CP} are the maximum inter-story drift ratios at the performance levels, respectively.

The non-performance probability, P_f, is defined as a function of the limit state functions corresponding to a given performance level. Estimation of the non-performance probability in the time-invariant domain requires the evaluation of the multiple integral over the failure domain, $G(U) < 0$, as follows [23]:

$$P_f = \iint \dots \int F_U(U)\, dU \tag{8}$$

where $F_U(U)$ is the joint probability density function of U.

As in the present work, only one limit state function is defined for each performance level, the total exceedence probability, Pf_E, for each performance level is defined as follows:

$$PF_E^i = P(G^i(U) \leq 0)\,, i = IO; LS; CP \tag{9}$$

Calculating the total exceedence probability, PF_E^i, requires the integration of a multi-normal distribution function [23]. However, this integral can be estimated by the MCS method. In this study, the MCS method is utilized simultaneously for all limit state functions of the performance levels. The MCS method allows the determination of an estimate of PF_E^i, given by:

$$PF_E^i = \frac{1}{n}\sum_{j=1}^{n} A_j^i(U)\,, i = IO; LS; CP \tag{10}$$

$$A_j^i = \begin{cases} 1.0 & if \quad G_j^i(U) \leq 0 \\ 0.0 & if \quad G_j^i(U) > 0 \end{cases}, i = IO; LS; CP \tag{11}$$

where n is the number of independent samples generated based on the probability distribution for each random variable for the MCS implementation.

Implementation of the MCS requires a large number of structural nonlinear analyses. The MCS is a time consuming process because of high computational cost of pushover analysis. To reduce the computational burden of MCS, a FFMLP NN model is trained to predict the required structural seismic responses.

3.2 FFMLP NN model
The FFMLP model is trained with back propagation (BP) technique, which is a gradient descent optimization algorithm that adjusts the weights in the steepest descent direction according to the following equation:

$$W_{t+1} = W_t - \eta \nabla_t \tag{12}$$

where W_t, ∇_t and η_t are the weight matrix, the current gradient matrix learning rate,

respectively at iteration t.

the BP technique uses Levenberg-Marquardt (LM) [14] algorithm to approach second-order training speed without having to compute the Hessian matrix. In the LM algorithm the weights updating is achieved as follows:

$$W_{t+1} = W_t - [J^T J + \alpha I]^{-1} J^T Er \tag{13}$$

where J is the Jacobian matrix that contains first derivatives of the network errors with respect to the weights; Er is a vector of network errors; α is a correction factor; and I is identity matrix.

One of the techniques used to prevent overfitting is regularization [14] in which the performance function of the network is modified by adding a term that consists of the mean of the sum of squares of the network weights as:

$$mse_r = \gamma \left(\frac{1}{m} \sum_{k=1}^{m} (Er_k)^2 \right) + \frac{1-\gamma}{nw} \sum_{l=1}^{nw} \left(W_{t,l} \right)^2 \tag{14}$$

where γ and nw are the performance ratio and number of network weights, respectively; m is the size of Er_k.

The input vector of the FFMLP model trained in the current study is the vector of random variables U and the components of its output vector are predicted maximum inter-story drifts at performance levels. The total number of 15 hidden layer neurons with tangent sigmoid transfer function are considered and the architecture of the network is shown in Fig. 2.
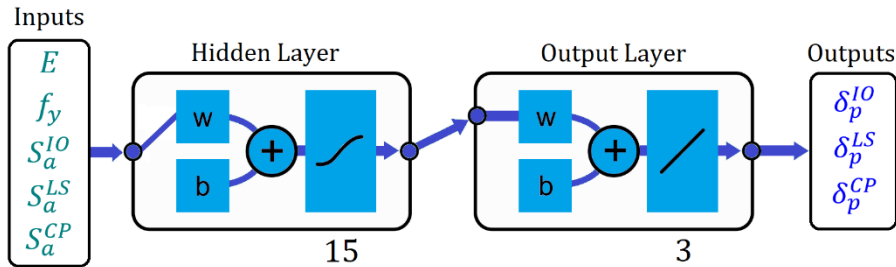

Figure 2. Architecture of the FFMLP model

To evaluate the prediction accuracy of the trained FFMLP NN model in training and testing modes, mean absolute percentage error (*MAPE*) between the ns number of actual (δ_{max}^i) and predicted (δ_p^i) responses is computed as follows:

$$APE_j^i = \left| \frac{\delta_{max}^i - \delta_p^i}{\delta_{max}^i} \right|_j , \quad i = IO; LS; CP , \quad j = 1,2,\dots,ns \tag{15}$$

$$MAPE^i = \frac{100}{ns} \sum_{j=1}^{ns} APE_j^i , \quad i = IO; LS; CP \tag{16}$$

4. NUMERICAL RESULTS

Three illustrative examples including 5-, 10-, and 15-story SCBFs with fixed and optimal topology of braces are selected from [24] in which these structures have been optimally designed in the context of PBD. The SCBFs with discrete fixed topology are denoted by DFT the structures with discrete optimal topology are denoted by DOT. The MCS-based reliability analysis of the optimally designed SCBFs is carried out using the probability density function, mean value and standard deviation of random parameter given in Table 1.

Table 1: Properties of random variables

Random Variable	Probability density function	Mean value	Standard deviation
E	Normal	200 GPa	20.0 GPa
f_y	Normal	345 MPa	34.5 MPa
S_a^{IO}	Lognormal	S_a^{IO}	$0.15 \times S_a^{IO}$
S_a^{LS}	Lognormal	S_a^{LS}	$0.15 \times S_a^{LS}$
S_a^{CP}	Lognormal	S_a^{CP}	$0.15 \times S_a^{CP}$

4.1 First example: 5-storey SCBF

All the 5-story SCBFs, optimally designed in [24], in DFT and DOT design groups are shown in Fig. 3.
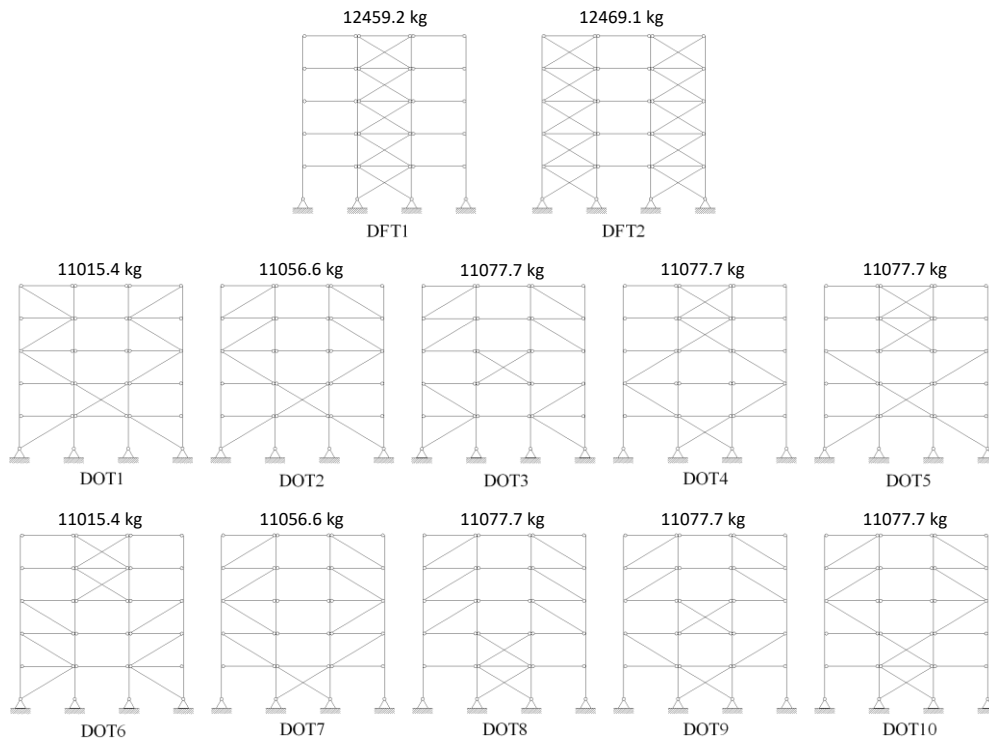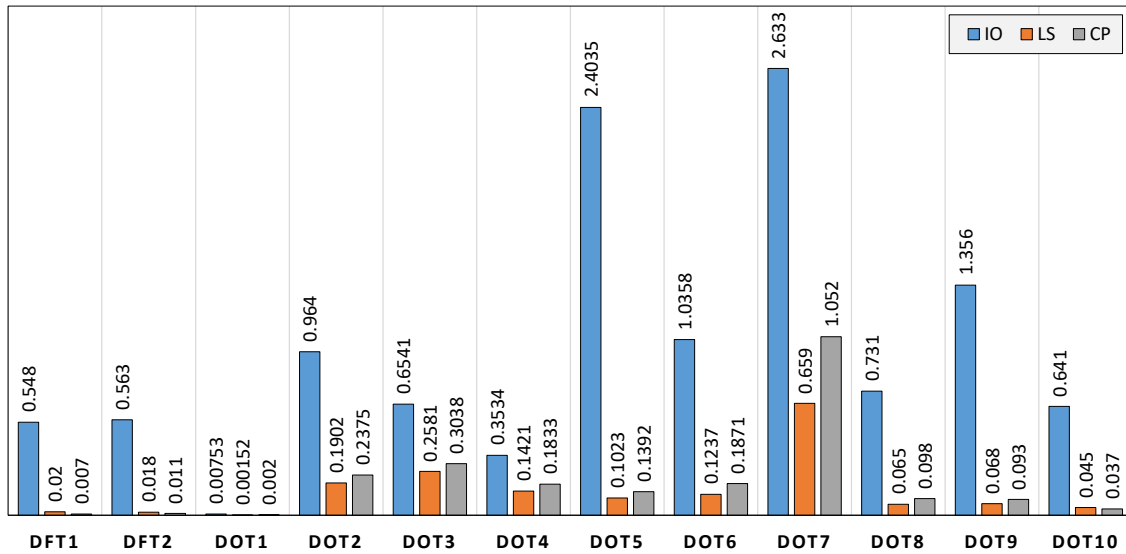

Figure 3. Optimally designed 5-story SCBFs

For each optimally designed 5-story SCBF, a FFMLP neural network is trained to predict the required seismic responses. A total number of 10,000 samples are generated and 8,000 and 2,000 samples are used for training and testing, respectively. *MAPE* values in training and testing modes are given in Table 2 for all the structures. These results show that the trained NN models have acceptable prediction accuracy.

Table 2: MAPE of different FFMLP NNs trained for 5-story SCBF

SCBF	δ_p^{IO}		δ_p^{LS}		δ_p^{CP}	
	Training	Testing	Training	Testing	Training	Testing
DFT1	9.23	9.53	3.33	3.02	2.19	2.61
DFT2	9.24	9.49	3.32	3.30	2.14	2.15
DOT1	10.91	10.53	3.84	3.81	2.64	2.65
DOT2	10.94	11.36	3.90	4.04	2.72	2.62
DOT3	9.91	9.71	3.55	3.49	2.92	2.79
DOT4	11.75	11.09	4.08	4.18	3.06	3.05
DOT5	10.39	10.49	3.45	3.47	2.02	2.08
DOT6	10.38	10.04	3.38	3.34	2.40	2.46
DOT7	7.33	11.68	2.74	3.11	2.13	1.92
DOT8	9.48	9.37	3.28	3.15	2.17	2.24
DOT9	11.5	11.05	3.52	3.37	2.25	2.29
DOT10	10.86	10.60	3.63	3.75	2.35	2.33

Reliability analysis of all the optimally designed 5-story SCBF is performed by using MCS method and the trained FFMLP NN models considering $n=10^6$ samples. The values of PF_E (%) obtained for all the 5-story SCBFs at performance levels are compared in Fig. 4.

Figure 4. PF_E (%) for optimally designed 5-story SCBFs at IO, LS and CP performance levels

The results of reliability assessment show that the highest safety against uncertainties belongs to the DOT1 structure which its PF_E is 0.0075%.

4.2 Second example: 10-storey SCBF

All the 10-story SCBFs, optimally designed in [24], in DFT and DOT design groups are shown in Fig. 5.
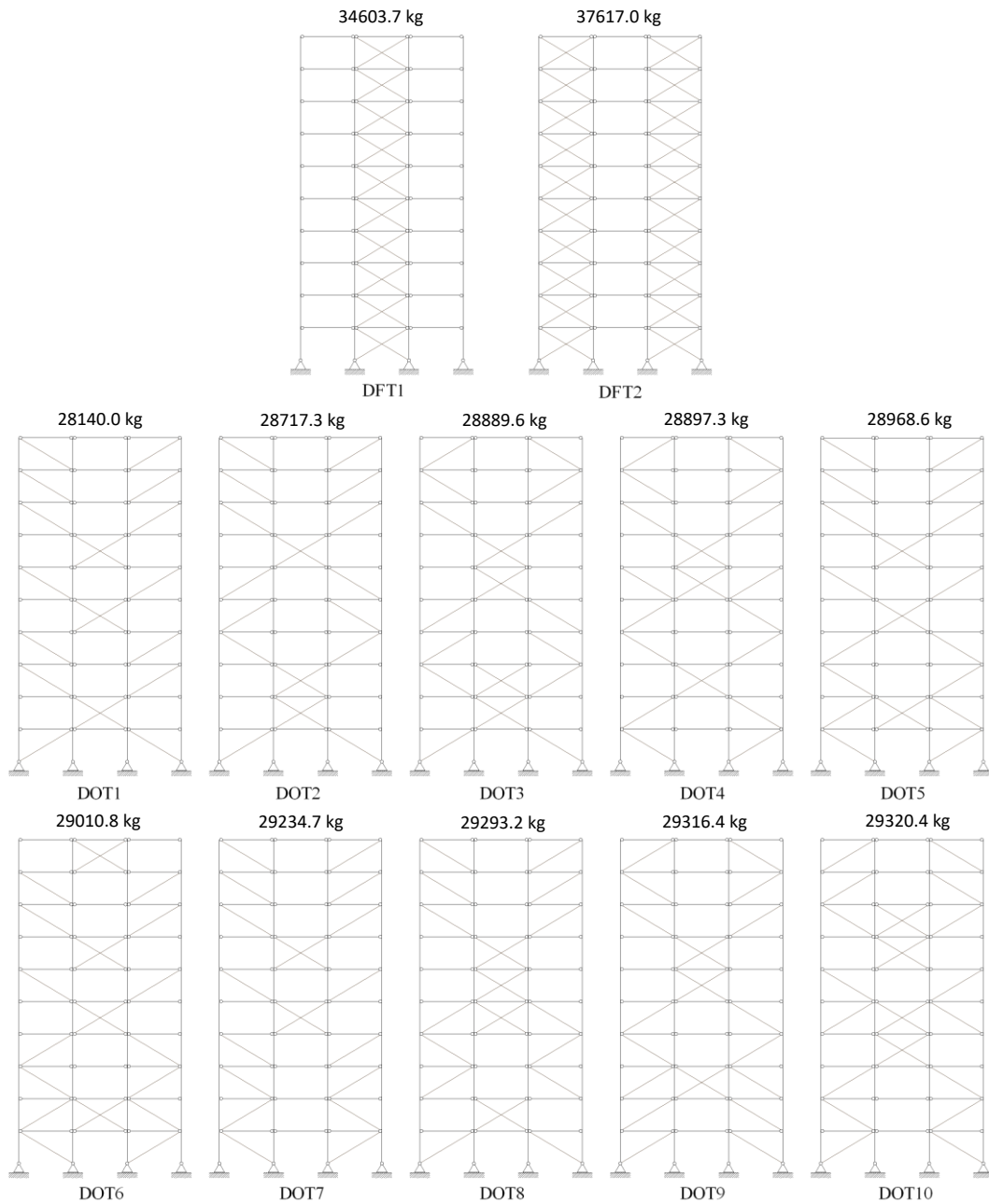

Figure 5. Optimally designed 10-story SCBFs

A FFMLP neural network is trained to predict the seismic responses of each optimally designed 10-story SCBF. A total of 10,000 samples are produced and 8,000 and 2,000 samples are used for training and testing, respectively. Table 3 reports the *MAPE* values in training and testing modes for all the structures indicating that the trained NN models have acceptable prediction accuracy.

Table 3: MAPE of different FFMLP NNs trained for 10-story SCBF

SCBF	δ_p^{IO}		δ_p^{LS}		δ_p^{CP}	
	Training	Testing	Training	Testing	Training	Testing
DFT1	4.63	4.76	4.90	4.84	7.56	7.44
DFT2	6.67	6.62	3.33	3.33	6.57	6.82
DOT1	7.29	7.42	4.13	4.29	7.26	7.92
DOT2	7.66	7.62	3.59	3.56	5.56	5.63
DOT3	8.73	8.16	3.48	3.38	3.18	3.19
DOT4	9.39	9.19	3.52	4.06	3.81	3.98
DOT5	8.04	7.83	3.26	3.14	2.95	2.91
DOT6	8.58	8.44	3.42	3.35	3.77	3.45
DOT7	6.54	6.24	4.17	3.60	7.91	7.42
DOT8	8.07	8.05	3.36	3.38	3.39	3.42
DOT9	8.91	8.67	3.73	3.75	3.98	3.76
DOT10	9.28	9.15	3.86	4.25	4.50	4.87

Reliability analysis of all the optimally designed 10-story SCBF is performed by using MCS method and the trained FFMLP NN models considering $n=10^6$ samples. The values of PF_E (%) obtained for all the 10-story SCBFs at performance levels are compared in Fig. 6.
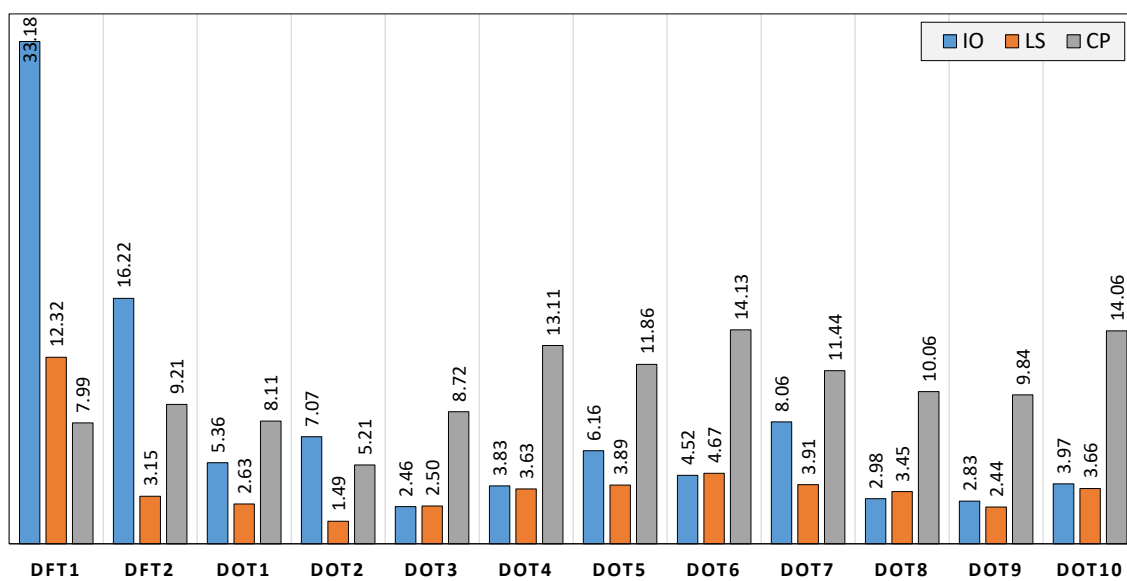

Figure 6. PF_E (%) for optimally designed 10-story SCBFs at IO, LS and CP performance levels

The results of reliability assessment show that the highest safety against uncertainties belongs to the DOT2 structure which its PF_E is 7.07%.

4.3 Third example: 15-storey SCBF

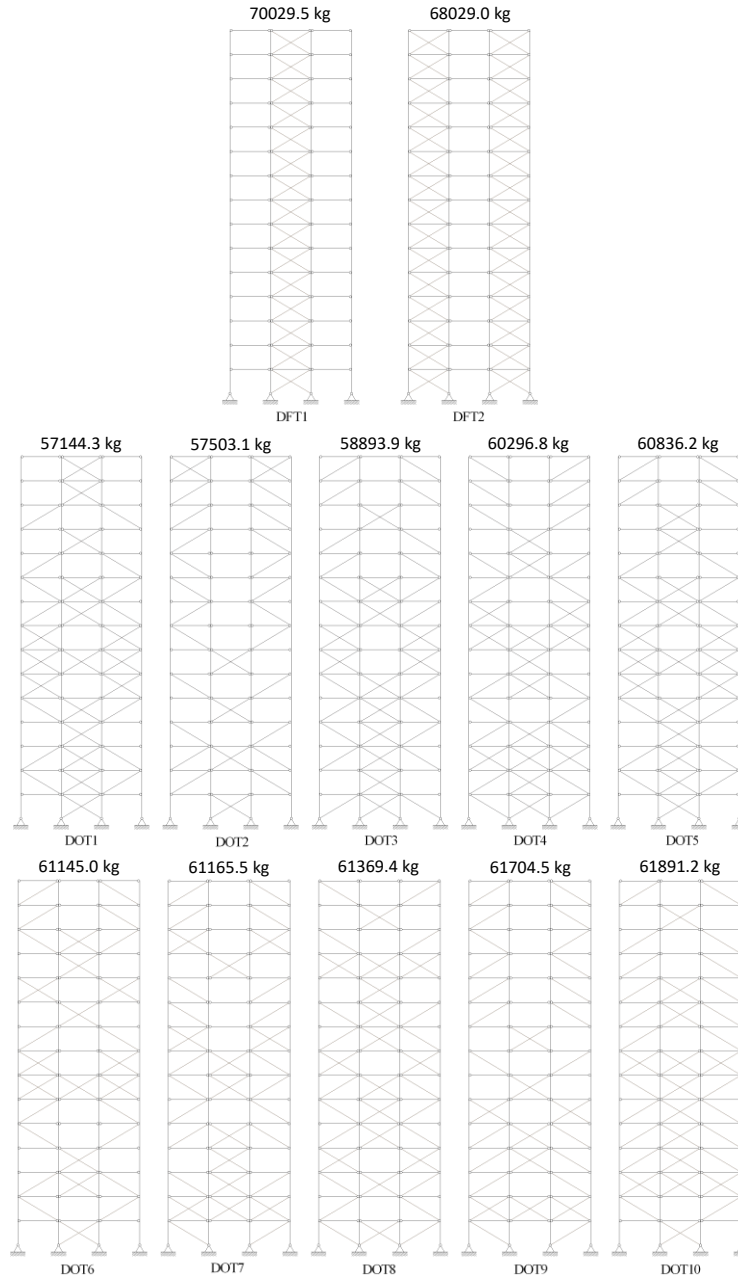All the 15-story SCBFs, optimally designed in [24], in DFT and DOT design groups are shown in Fig. 7.

Figure 7. Optimally designed 15-story SCBFs

In order to evaluate the seismic responses of optimally designed 15-story SCBF, A

FFMLP neural network is trained for each structure. A total of 10,000 samples are produced and 8,000 and 2,000 samples are used for training and testing, respectively. For all the structures, the *MAPE* values in training and testing modes are given in Table 4. As seen, the prediction accuracy of the trained NN models is acceptable.

Table 4: MAPE of different FFMLP NNs trained for 15-story SCBF

SCBF	δ_p^{IO}		δ_p^{LS}		δ_p^{CP}	
	Training	Testing	Training	Testing	Training	Testing
DFT1	5.08	5.07	3.90	3.97	8.40	7.94
DFT2	3.01	3.12	3.41	3.58	5.43	4.83
DOT1	5.59	3.61	4.72	3.09	6.45	3.50
DOT2	5.22	5.13	3.39	3.64	7.61	7.95
DOT3	4.95	4.74	3.65	3.27	8.60	8.26
DOT4	5.38	5.41	4.35	4.11	4.46	3.92
DOT5	6.62	6.44	5.65	5.03	8.43	9.69
DOT6	6.33	5.95	4.94	3.87	11.73	10.64
DOT7	5.61	5.47	4.65	4.78	10.87	10.86
DOT8	7.01	6.69	2.57	2.46	3.65	3.69
DOT9	4.61	4.56	2.70	2.59	4.16	4.22
DOT10	8.41	7.58	4.96	4.81	8.62	9.02

Reliability analysis of all the optimally designed 15-story SCBF is performed by using MCS method and the trained FFMLP NN models considering $n=10^6$ samples. The values of PF_E (%) obtained for all the 15-story SCBFs at performance levels are compared in Fig. 8.
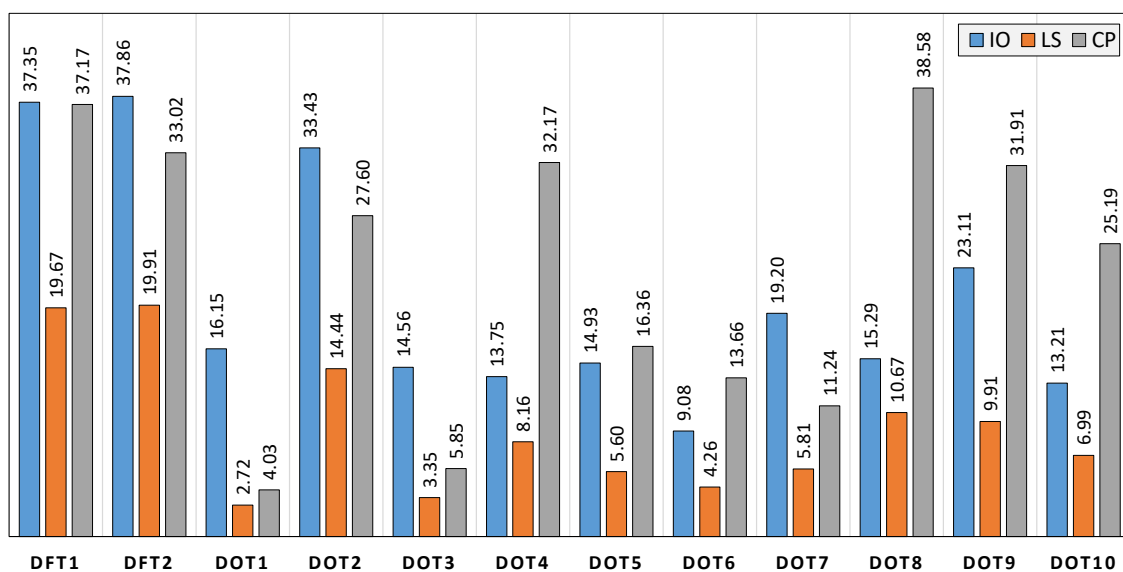

Figure 8. PF_E (%) for optimally designed 15-story SCBFs at IO, LS and CP performance levels

The results of reliability assessment show that the highest safety against uncertainties

belongs to the DOT6 structure which its PF_E is 13.66%.

5. CONCLUSIONS

Reliability analysis of optimally designed SCBF structures was conducted in the present study. For this purpose, a FFMLP NN incorporated MCS method was proposed. Three numerical examples of 5-, 10- and 15-story SCBFs were selected from [24] where 12 optimal designs have been obtained for each design example in the PBD framework. A total of 36 FFMLP NN model were trained in the current study for predicting the seismic responses of the optimally designed SCBFs in the framework of MCS. To perform the reliability analysis, limit state functions were defined on the maximum inter-story drift ratios at the IO, LS and CP performance levels. The main findings of this study were summarized as follows:

- The results of the reliability analyses demonstrate that the maximum value of PF_E for 5-, 10- and 15-story SCBFs in DOT design group is 2.63%, 14.13% and 38.58%, respectively.
- PF_E for 5-, 10- and 15-story optimally designed SCBFs is depicted in Fig. 9. It can be concluded that the seismic reliability of 15-story optimally designed SCBFs is questionable.
- Some SCBFs with optimal topology of braces are more reliable than those with fixed topology of braces.
- The minimum values of PF_E which show the safest structures against uncertainties are 0.0075%, 7.07% and 13.66% for 5-, 10- and 15-story SCBFs, respectively. The safest 5-, 10- and 15-story optimally designed SCBFs are shown in Fig. 10.
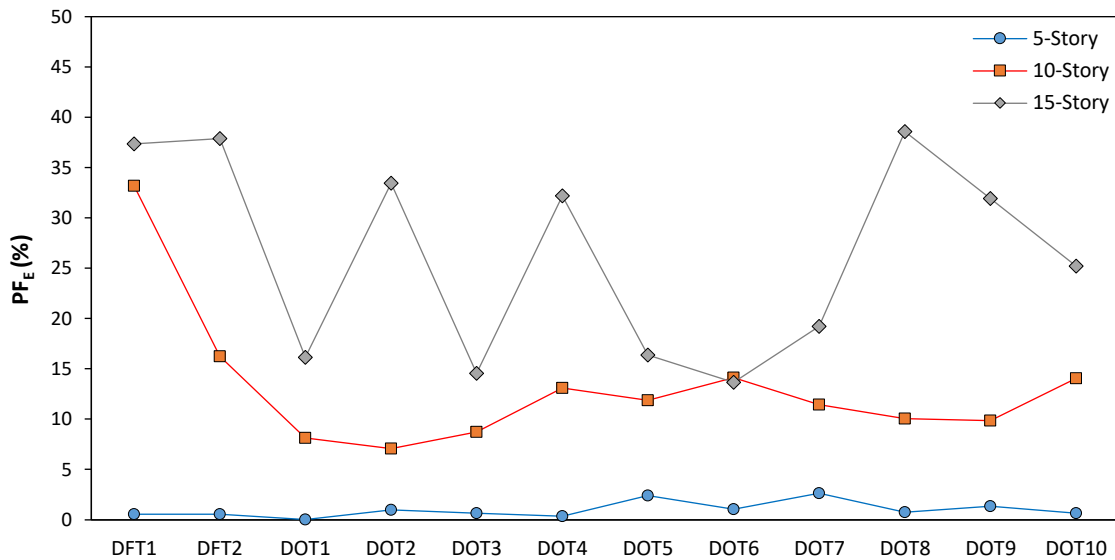

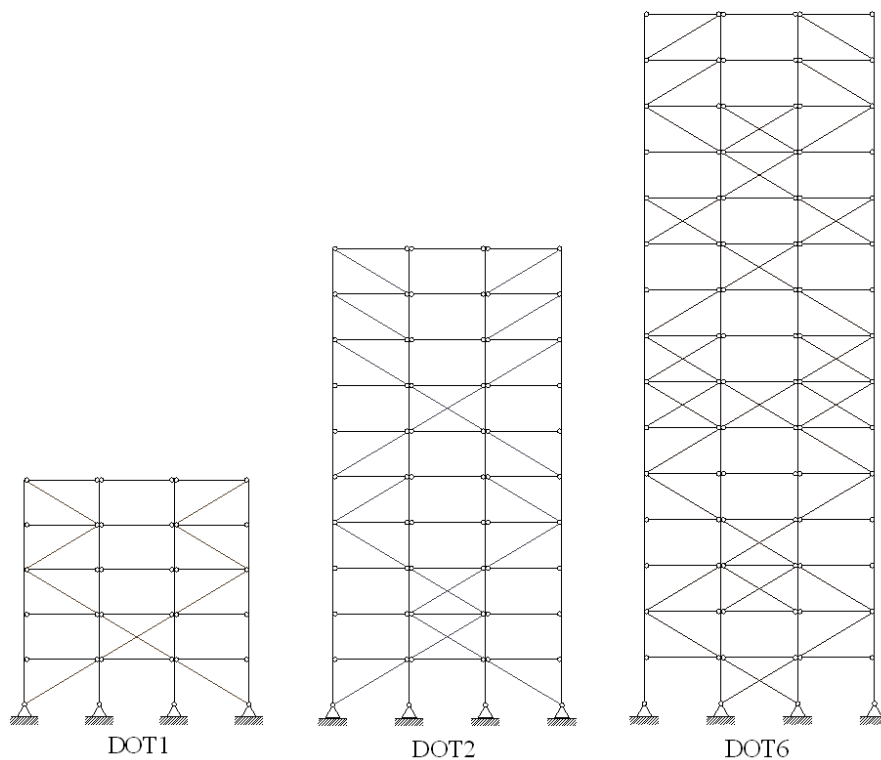Figure 9. PF_E (%) for optimally designed 5-, 10- and 15-story SCBFs

Figure 9. The safest optimally designed 5-, 10- and 15-story SCBFs

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