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STRUCTURAL DAMAGE IDENTIFICATION BASED ON CHANGES IN NATURAL FREQUENCIES USING THREE MULTI-OBJECTIVE METAHEURISTIC ALGORITHMS

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ABSTRACT

In order to evaluate the damage state, value, and position of structural members more accurately, a multi-objective optimization (MO) method is utilized that is based on changes in natural frequency. The multi-objective optimization dynamic-based damage detection method is first introduced. Two objective functions for optimization are then introduced in terms of changing the natural frequencies and mode shapes. The multi-objective optimization problem (MOP) is formulated by using the two objective functions. Three considered MO algorithms consist of Colliding Bodies Optimization (MOCBO), Particle Swarm Optimization (MOPSO), and non-dominated sorting genetic algorithm (NSGA-II) to achieve the best structural damage detection. The proposed methods are then applied to three planar steel frame structures. Compared to the traditional optimization methods utilizing the single-objective optimization (SO) algorithms, the presented methods provide superior results.

Keywords: Damage detection; Natural frequency; Optimization algorithms; Multi-objective optimizations; frame structures.

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1. INTRODUCTION

For structural health monitoring, it is necessary to use non-destructive methods to obtain

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information about the presence. location, and extent of damages. Monitoring and interpreting changes in structural dynamic properties by measurements and the use of experimental modal analyses via signal-processing techniques is a usual method [1]. Modern vibration testing equipment and instrumentation can be utilized to extract the natural frequency and mode shape of a vibrating structural system. Optimization algorithms are powerful means to detect structural damages and can efficiently be used for data analysis to identify potential damage locations.

Several methods have been developed to find structural damages by measuring the change of the natural frequencies [2-11]. This problem can be formulated as a bounded nonlinear optimization problem and finding the best solution. The main idea is to alter the characteristic of numerical models to match the experimental data values, highlighting damaged regions and the extent of damage in the structure. An optimization algorithm aims at finding the optimal parameters values, which are the reduction factors of element stiffness, to achieve a pre-defined performance based on the modal parameters outlined by the experimental data. This process results in a target performance optimization problem, which is highly challenging to solve due to the presence of non-convex and multimodal objective functions. The deterministic optimization algorithms may not converge to the global minimum of the problem due to their dependence on the quality of the starting point of the search. With the rapid development of the computation technologies, the problem has been solved using recent metaheuristics such as the genetic algorithm (GA) the bee algorithm (BA) [12], the PSO algorithm [19], the HS algorithm [20], and the improved CSS algorithm [21] and the CBO algorithm [13]. However, all the above methods were based on the optimization of a single objective optimization, which is defined as the difference between measured and theoretical output data. The structural complexity makes it impossible for these algorithms to identify the damage in these works. To improve the search capability, it is necessary to adopt strategies that increase the efficiency.

This study presents the damage detection of frame structures based on changes in the natural frequencies using muti-objective optimization algorithms (MOAs). The main contributions of this research work are as follows:

(i) This study introduces an extension of MOA to damage detection in frame structures through numerical examples;
(ii) This problem is solved using the recent MOAs and a comparative study is carried out;
(iii) A comparison is made between the results achieved using MOAs and the traditional strategy based on SOAs.

The present paper is organized as follows: In the next section, the problem formulations of detecting damage in structures is provided. The optimization algorithms are then explained, followed by a section consisting of the study of three damage detection problems of frame structural. Conclusions are derived in the last section.

2. PROBLEM FORMULATION

The structural modal property is an important indicator for structural damage identification. To depict the damaged state of a structure, the simplest way is to measure the stiffness properties of a structure. The change in mass is expected to be insignificant compared to stiffness [6]. The global stiffness matrix of the damaged structure ($[k^d]$) can be obtained as the summation of the damaged and undamaged element stiffness matrices ($[k_e]$), where the reduction factor (α_i) multiplies the stiffness of the damaged local element ($[k_e^d]$), such as:

$$\left[K^d\right]=\sum_{i=1}^{N}\left[K_e^d\right]_i=\sum_{i=1}^{N}(1-\alpha_i)\left[K_e\right]_i \tag{1}$$

Where N is the number of structural elements. The reduction factor (α_i) indicates the severity of the damage in the ith element of the finite element model whose values are between 0 for elements having no damage and 1 for ruptured elements. The factors α and (1-α) are also defined as the damage and health severity, respectively. Moreover, it is assumed that no change occurs after damage in the mass matrix $[M]$, which seems to be reasonable in most of the real problems.

The jth eigenvalue equation of the damaged structure can be generated by replacing the structure's stiffness matrix by that of the damaged one:

$$\left[K^d\right]\{\varphi_{jd}\}-\omega_{jd}^2\left[M\right]\{\varphi_{jd}\}=\{0\} \tag{2}$$

in which, ω_{jd} and φ_{jd} are the jth natural frequency and the jth shape mode of the damaged structure, respectively.

In the single objective classic optimization problem, the objective function is expressed as the fractional changes in natural frequencies and mode shapes before and after damage:

$$F(\alpha)=\sum_{i=1}^{NM}\left(\left(\frac{\delta\omega_i(\alpha)}{\omega_i}\right)^D-\left(\frac{\delta\omega_i}{\omega_i}\right)^E\right)^2+\sum_{i=1}^{NM}\sum_{j=1}^{NP}\left(\left(\delta\varphi_{ij}(\alpha)\right)^D-\left(\delta\varphi_{ij}\right)^E\right)^2 \tag{3}$$

Where, NM is the number of analyzed modes, NP is the number of nodal displacement that is measured, the superscripts D and E represent numerical and experimental quantities, respectively, ω_i is the natural frequency for the ith mode of the undamaged state, $\delta\omega_i$ and $\delta\varphi_{ij}$ are fractional change of the experimental and analytical natural frequencies and displacement nodal for the ith mode of the structure, respectively. Until the differences in the numerical frequencies between healthy and damaged states converge to the observed experimental frequencies in the pre and post damaged states, the stiffness reduction factor (α) of the finite element model should be updated [7].

The problem mentioned earlier was solved by the SO algorithms, and the number of variables was the same as the number of structural elements. By increasing the number of variables, these algorithms prevented finding the best solution, particularly when dealing with frame structure damage detection problems. Then, it was essential to adopt a strategy to enhance the search space exploration in this optimization problem.

In this study, MO algorithms are utilized to detect damage in structures. MO algorithms aim to find stiffness reduction factors (α) for the finite element model that simultaneously minimize the objective functions defined by Eq. (4). These objective functions (F_1 and F_2) represent the differences in the natural frequency and node displacement between healthy and damaged states.

$$F_1(\alpha)=\sum_{i=1}^{NM}\left(\left(\frac{\delta\omega_i(\alpha)}{\omega_i}\right)^D-\left(\frac{\delta\omega_i}{\omega_i}\right)^E\right)^2$$
$$F_2(\alpha)=\sum_{i=1}^{NM}\sum_{j=1}^{NP}\left(\left(\delta\phi_{ij}(\alpha)\right)^D-\left(\delta\phi_{ij}\right)^E\right)^2 \tag{4}$$

The objective functions in MO algorithms must be in conflict with each other, such as when one objective function increases, the other decreases. In this case, the user is given a set of solutions (referred to as the Pareto front) by multi-objective algorithms. The objective functions employed in this research are in accordance with one another, and they are only used to find the optimal point in the problem search space as accurately as possible. The purpose of the damage detection is to find a single solution that shows the structure's damage status. Hence, if the multi-objective algorithm only gives one solution, that point is the optimal one. Otherwise, the best solution in the Pareto front will be chosen using the Knee point method described in Reference [18]. Figure 1 displays the flow chart of the proposed method.

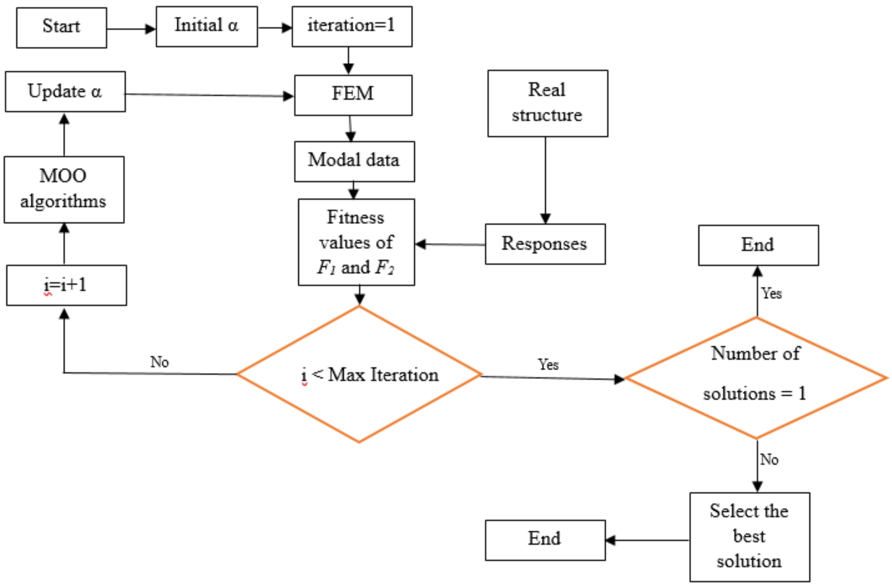


Figure 1. The general flowchart of the proposed method

3. OPTIMIZATION ALGORITHMS

As mentioned before, the MOCBO, MOPSO and NSGA-II algorithms have been employed for solving Eq. (4). In this section, these algorithms are briefly presented.

3.1 Multi-Objective Colliding Bodies Optimization Algorithm

Algorithm 1 is named 'multi-objective colliding bodies optimization' (MOCBO) because it utilizes the CBO formulation for the search process. This algorithm aims to find optimal solutions for multiple objective optimization problems by simulating the interactions and collisions between bodies representing potential solutions [14,15].

Algorithm 1 Multi-objective colliding bodies optimization (MOCBO)

Initialize
- Generate an initial population of candidate solutions (bodies)
- Evaluate the objectives for each body
- Initialize the non-dominated archive

Begin
- **While** (termination condition is met) **do**
  - Evaluate the maximin value of each solution
  - Arrange the populations
  - Divide the populations inti two equal groups
  - Determine the collision between bodies
  - Update the positions of the bodies
  - Update the non-dominated archive
- **End While**

End

3.2 Multi-objective Particle Swarm Optimization Algorithm

In the field of multi-objective optimization, MOPSO is a metaheuristic optimization algorithm that has been widely utilized. In the MOPSO a population of candidate solutions called particles iteratively searches for optimal solutions by moving through the search space based on the best solutions so far found [16]. The evaluation of solution quality by using Pareto's dominance is the main concept behind MOPSO. The MOPSO algorithm maintains track of both the global and personal best position for every particle (Algorithm 2).

Algorithm 2 Multi-objective particle swarm optimization (MOPSO)

Initialize
- Initialize the population of particles
- Assign random velocities to each particle
- Evaluate the objectives for each particle
- Initialize the non-dominated archive

Begin

While (stopping criteria are NOT satisfied) **do**

For each particle

Update the personal best solution (pbest)

Update the global best solution (gbest)

Update the velocity and position of each particle

End For

Update the non-dominated archive

End While

End

3.3 Non-dominated Sorting Genetic Algorithm-II

The Non-dominated Sorting Genetic Algorithm-II, known as NSGA-II, is a commonly utilized multi-objective optimization technique that aims to discover a set of non-dominated solutions in a given search domain [17]. NSGA-II works by maintaining a population of candidate solutions and evolving them over multiple generations. It uses techniques like non-dominated sorting diversity preservation and elitism to push the search process towards the Pareto-optimal front where no other solution is better among all objectives simultaneously.

In order to preserve diversity of the population, it evaluates individuals according to their dominant relationship with each other and assigns them ranks and crowding distances. By selecting the best individuals through a combination of dominance and diversity measures, NSGA-II is able to achieve an efficient approximation of the Pareto front (Algorithm 3).

Algorithm 3 Non-dominated Sorting Genetic Algorithm-II (NSGA-II)

Initialize

Generate an initial population of candidate solutions randomly

Evaluate the objective function values for each individual in the population

Initialize the non-dominated archive

Begin

While (termination condition is met) **do**

Calculate the crowding distance for individuals in each Pareto front

Select individuals for the next generation

Perform crossover and mutation operators on the selected individuals

Combine the current population and offspring to form a new population

Update the non-dominated archive

End While

End

It will be interesting to use force method in place of the displacement approach. To achieve this other application of the force method can be found in Refs. [22-27].

4. NUMERICAL EXAMPLES

In this section three planar frame structures are considered for the numerical study to evaluate the feasibility and effectiveness of the proposed methods. In addition, the SO algorithms have been employed in the first example to compare the results of the adopted method with classic methods. In all the examples, a total of 100 populations are used for all MO and SO algorithms. For the first, second and third examples, the maximum number of iterations is considered as 200, 400 and 400, respectively. The programming language used for writing the algorithms is Matlab. In all of the examples, two damage scenarios are utilized as elements that decrease Young's modulus. The first and second damage scenarios are described as single and multiple damage cases, respectively.

4.1. Example 1: A portal plane frame

A planar steel frame with 56 equal-length beam elements and 57 nodes that is considered the first example as shown in Figure 2. The material density is taken as $2500kg/m^3$ and the modulus of elasticity is considered as 25,000 *MPa*. The structure is rectangular in shape with a cross-sectional area of *h*=0.24m high and *b*=0.14m wide [9]. Table 1 shows that the reduction factor determines the damage severity of each element in both damage scenarios of this example: (1) 20% damage in element 1, (2) 15% damage in element 10 and 20% damage in element 22. The objective functions employed in this example are described by Equation (3) and Equation (4) for the SO and MO algorithms, respectively. The mode shapes in these equations incorporated the initial 16 natural frequencies and all the nodal displacements.

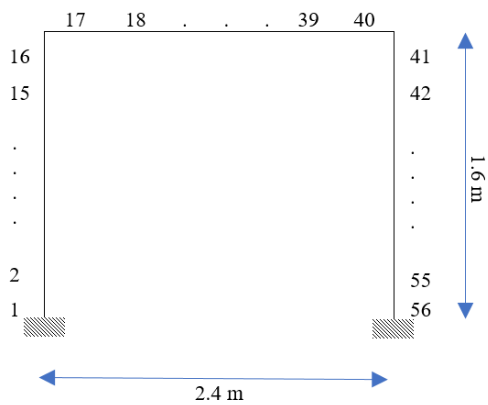


Figure 2. A portal plane frame

Figure 3 shows the predicted health severities (1-α) in elements using the SO algorithms. It can be seen that these algorithms cannot solve this problem and makes a mistake in finding the damage in structural elements

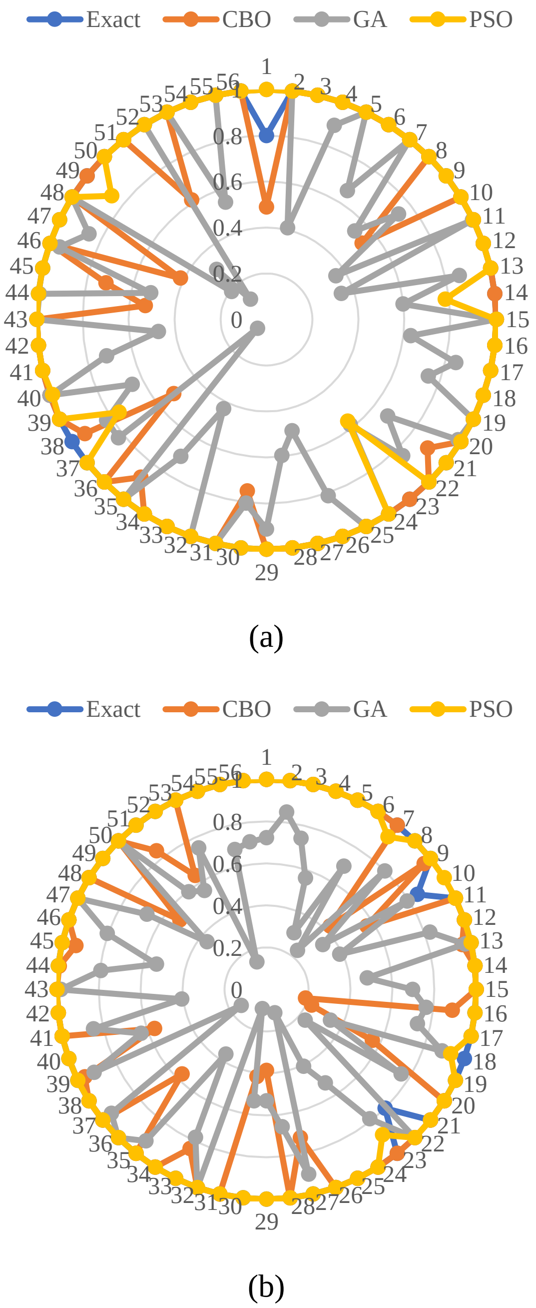
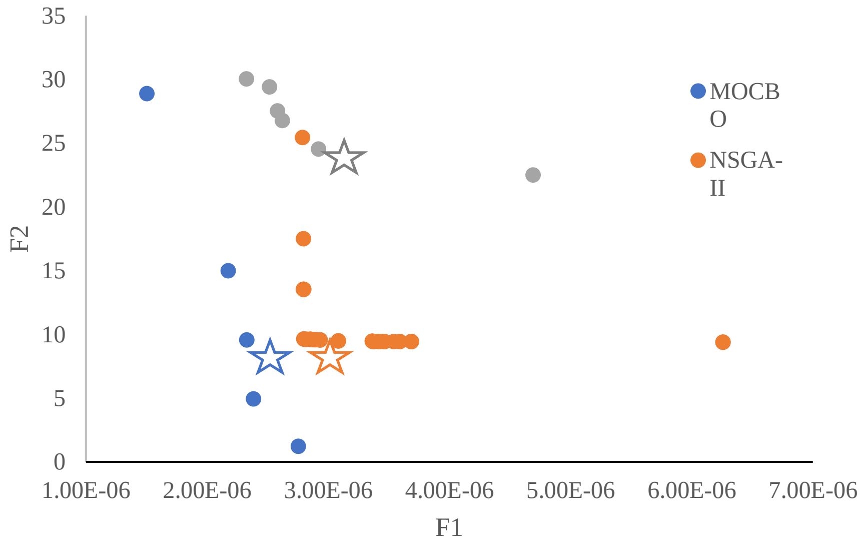


Figure 3. The obtained damage severities of SO algorithms for the first example: a) scenario 1, b) scenario 2

Figure 4 shows the trade-off between the objective functions introduced in Eq. (4) using three multi-objective optimization methods. Since the PF sets have more than one solution, the best solution is chosen using the Knee point method described in Ref. [18]. The best solutions found using MO algorithms are also displayed in Figure 4 with a star symbol. The PF set obtained through MOCBO is dominated by those obtained through MOPSO and NSGA-II, as can be observed. Table 1 displays the best solutions found in this example using MO algorithms. Figure 5 also illustrates the most accurate predictions of the amount of damage occurs in different elements using MO algorithms. By comparison, one can see that the predicted severity damage using MOCBO is closer to the exact value compared to the outcome of the NSGA-II and MOPSO.



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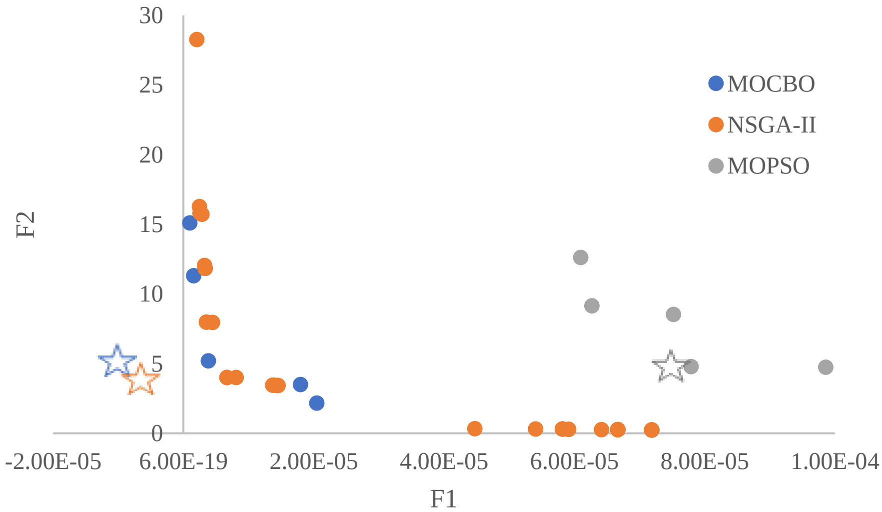
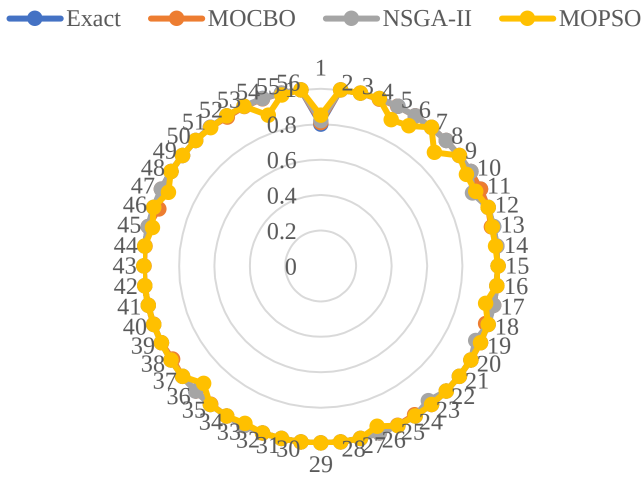


(b)

Figure 4. The obtained Pareto Front of MO algorithms for the first example: a) scenario 1, b) scenario 2 (Stars represent the best solutions in PF sets)

Table 1. Results obtained by different MO algorithms for both damage scenarios in the first example

	Damage location	**Exact severity damage (%)**	**Predicted severity damage (%)**		
			MOCBO	MOPSO	NSGA-II
Scenario 1	1	20	19.186	14.870	18.163
Scenario 2	10	15	18.886	23.365	19.191
	22	20	23.541	28.642	25.853



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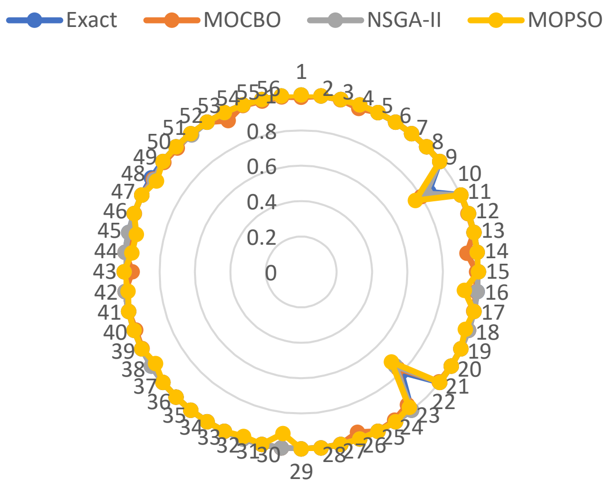


(b)

Figure 5. The obtained health severities of MO algorithms for the first example: a) scenario 1, b) scenario 2

4.2. Example 2: A two-bay and three-story plane frame

A two-bay and three-story plane steel frame, as shown in Figure 6, with a 36-element model consisting of nine columns and six beams, and 30 free nodes is examined as the second example. The structural beams are divided into three equal elements and the columns are divided into two equal elements. The frame structure model is made up of 36 two-dimensional beams. The material density is 7708 kg/m^3 and the modulus of elasticity is 207 GPa. The moment of inertia, and cross-sectional area of the all elements are considered as I=3.3×10^4 m^4 and A=1.5×10^2 m^2, respectively.

In this example, the severity of the damage in each element is shown by the reduction factor in Table 2 for both damage scenarios: (1) 10% damage in element 6, (2) 15% damage in element 10 and 20% damage in element 22. In this example, the first 15 natural frequencies and all the nodal displacements in the mode shapes are used.

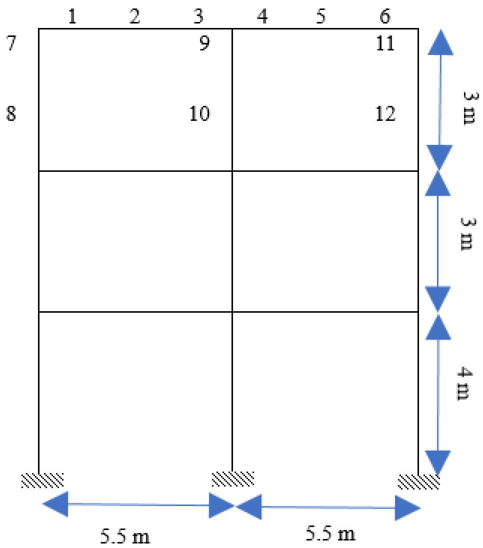
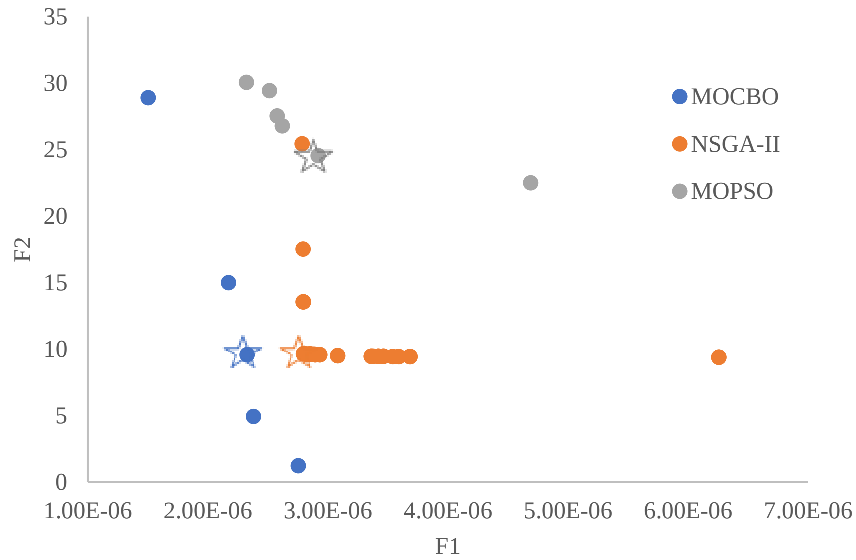


Figure 6. A 2-bay and 3-story frame with the finite-element model

Table 2. Results obtained by different MO algorithms for both damage scenarios in the second example

	Damage location	**Exact severity damage(%)**	**Predicted severity damage(%)**		
			MOCBO	MOPSO	NSGA-II
Scenario 1	2	40	39.165	5.466	36.625
Scenario 2	2	10	10.006	14.852	10.553
	5	20	19.101	19.151	19.590

The trade-off's curves and best solutions obtained in this example using three provided multi-objective optimization algorithms are shown in Figure 7. It is clear that the PF set from MOCBO is dominated by the ones from MOPSO and NSGA-II. Table 2 displays the best solutions found in this example using MO algorithms. Figure 8 shows the predicted health severities (1- α) in the elements using the proposed MO algorithms. The most precise estimates of how much damage will be involved in various elements by the MO algorithms are also presented in Figure 8. Similar to the first example, one can see that MOCBO predicts the damage severity better than NSGA-II and MOPSO.



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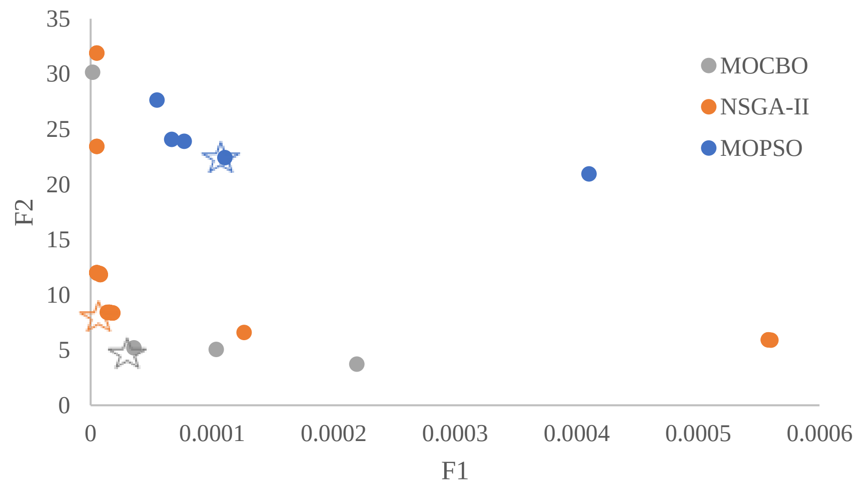
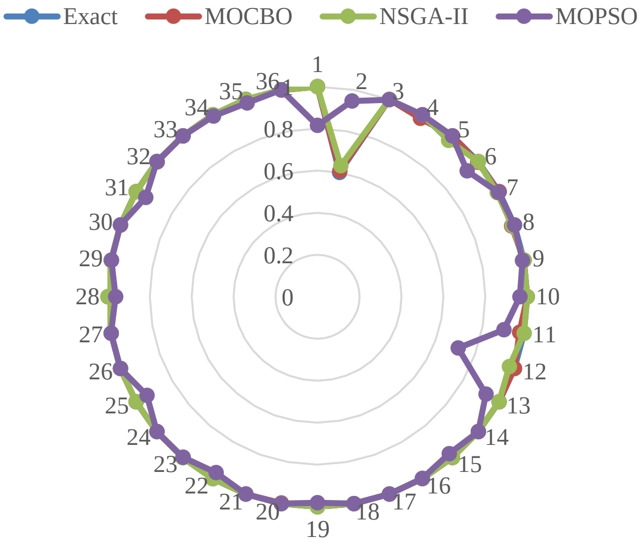


(b)

Figure 7**.** The obtained Pareto Front of MO algorithms for the second example: a) scenario 1, b) scenario 2 (Stars represent the best solutions in PF sets)

4.3. Example 3: A 5-bay and 10-story plane frame

A five-bay and ten-story plane steel frame, as depicted in Figure 9, with a 110-element model consisting of 60 columns and 50 beams, and 60 free nodes is examined as the last example. The frame structure model is made up of 110 two-dimensional beams. The material density is 2500 kg/m^3 and the modulus of elasticity is 250 GPa. The sections of columns and beams from the first to the fifth floors are considered BOX 35×35×1 and IPB300, respectively. The sections of columns and beams from the sixth to the tenth floors are considered BOX30×30×1 and IPB280, respectively.



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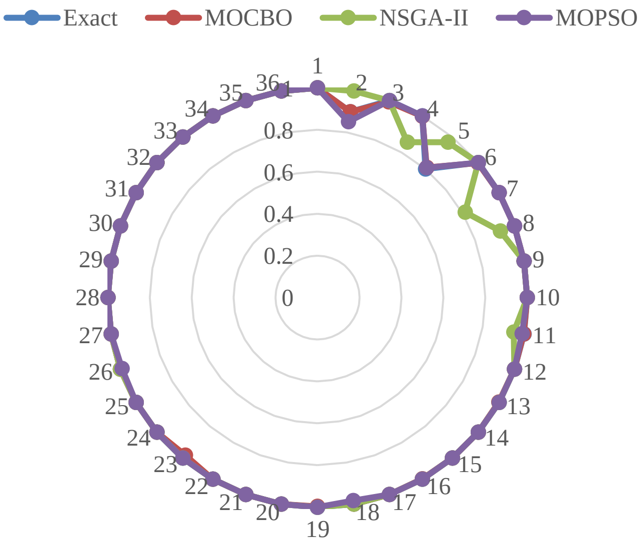


(b)

Figure 8. The obtained health severities of MO algorithms for the second example: a) scenario 1, b) scenario 2

In this example, the severity of the damage in each element is shown by the reduction factor in Table 3 for both damage scenarios: (1) 15% damage in element 108, (2) 5%

damage in element 101 and 15% damage in element 107. In this example, the first 20 natural frequencies and all the nodal displacements in the mode shapes are used.

Table 3. Results obtained by different MO algorithms for both damage scenarios in the third example

	Damage location	Exact severity damage(%)	Predicted severity damage(%)		
			MOCBO	MOPSO	NSGA-II
Scenario 1	108	15	13.463	20.770	0
Scenario 2	101	5	4.610	31.244	15.643
	107	15	13.493	0	0

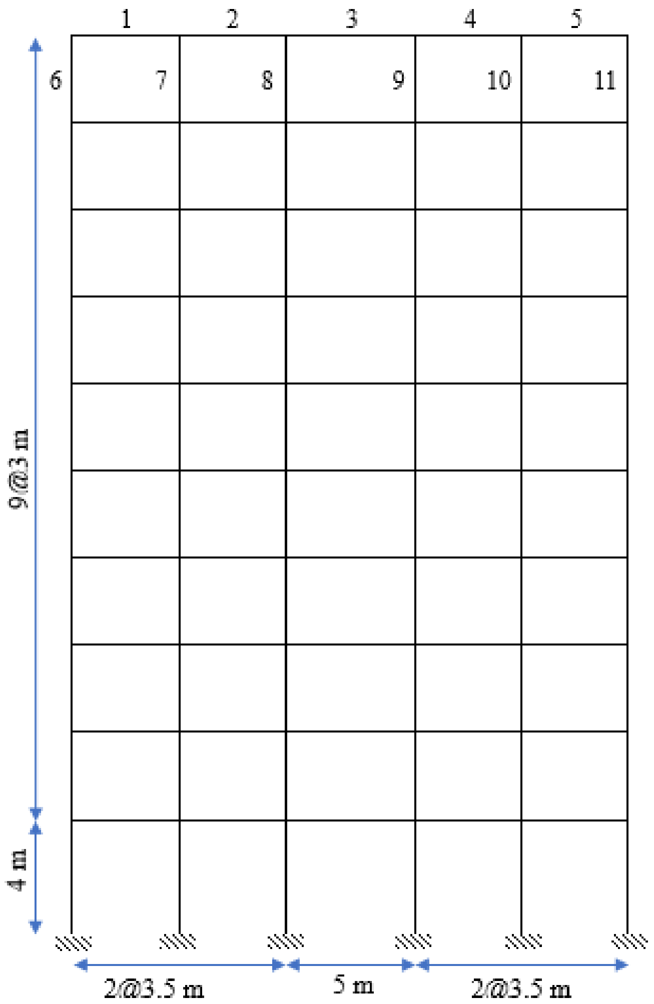
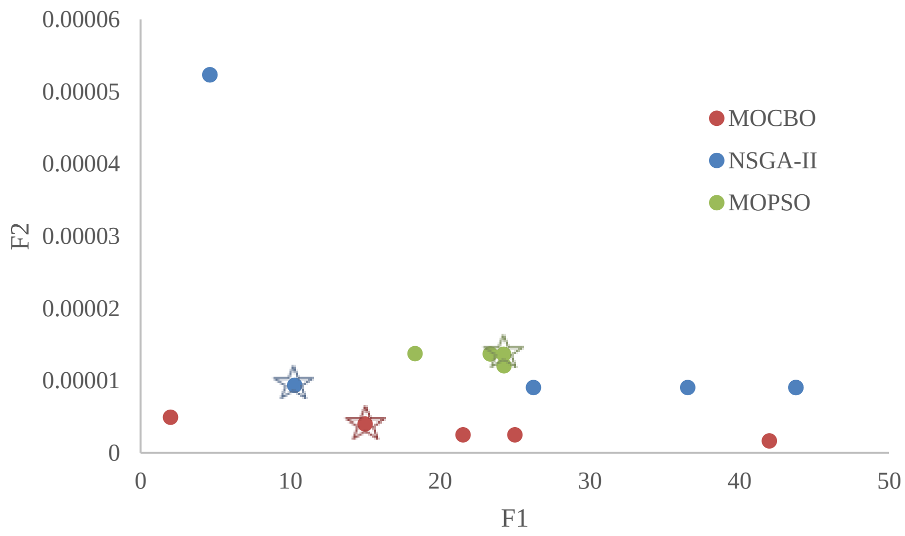


Figure 9. A 5-bay and 10-story frame with the finite-element model

The trade-off's curves and best solutions obtained in this example using three provided multi-objective optimization algorithms are shown in Figure 10. It is clear that the PF set from MOCBO is dominated by the ones from MOPSO and NSGA-II. Table 3 displays the best solutions found in this example using MO algorithms. Figure 11 shows the predicted health severities (1- α) in the elements using the proposed MO algorithms. The most precise estimates of how much damage will be done to various elements by MO algorithms are also presented in Figure 11. Similar to the first and second examples, one can see that MOCBO predicts the damage severity better than NSGA-II and MOPSO.



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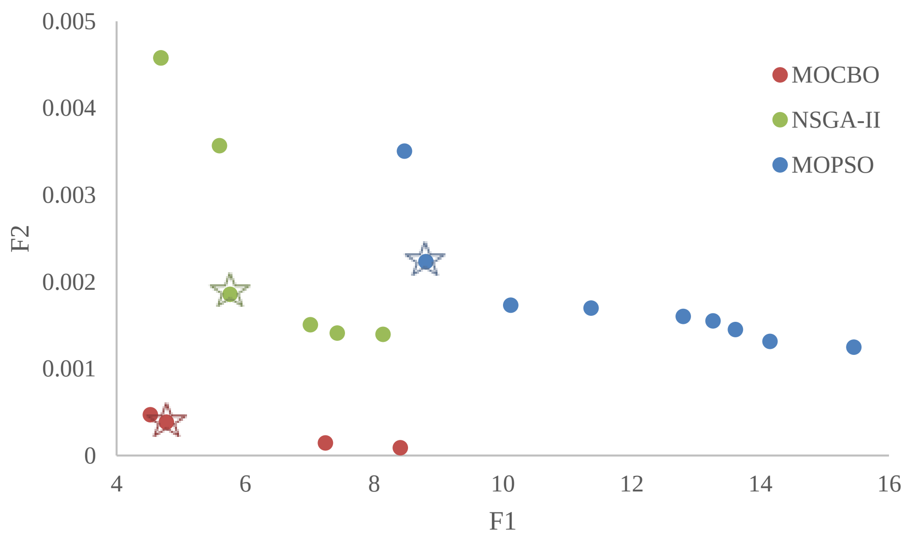

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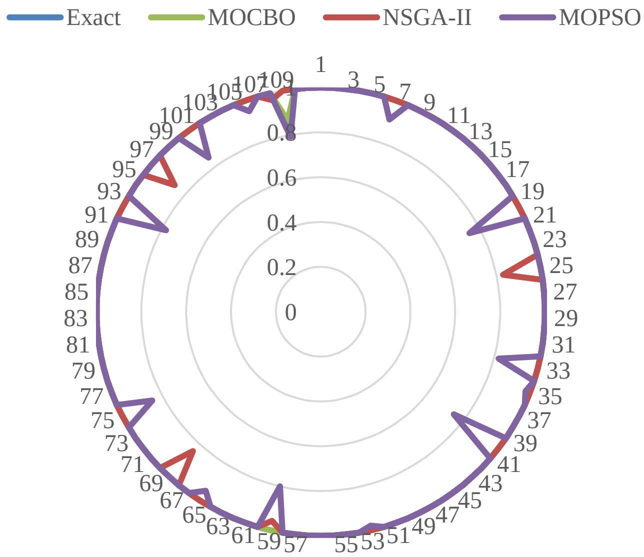
Figure 10. The obtained Pareto Front of MO algorithms for the third example: a) scenario 1, b) scenario 2 (Stars represent the best solutions in PF sets)



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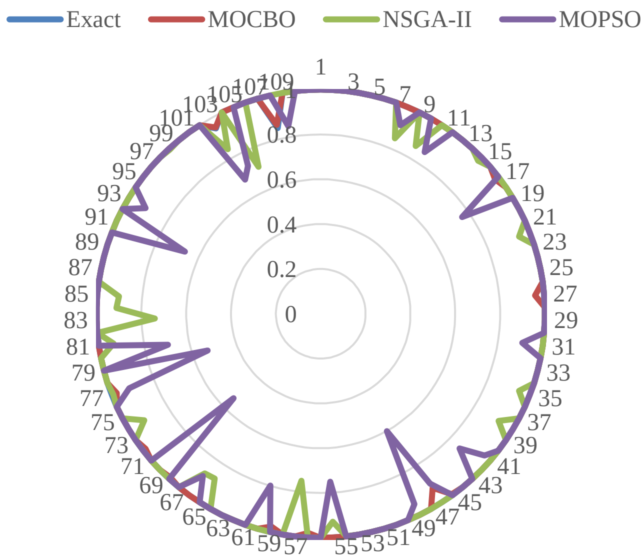

(b)

Figure 11. The obtained health severities of MO algorithms for the third example: a) scenario 1, b) scenario 2

4. CONCLUDING REMARK

In this study, three multi-objective meta-heuristic algorithms are presented consisting of the MOCBO, MOPSO and NSGA-II, for damage detection of planar frame structures using changes in natural frequencies. Initially, objective functions are defined for finding the location and qualification of damage severity of structural elements based on modal information of the damaged structures. Then, the MO algorithms are used to determine the damage in structures by optimizing the cost functions.

The efficiency of the proposed algorithms is investigated with three simple frame structures. The performances of these examples are compared using noise-free modal data through simulated damage scenarios. Two damage scenarios, namely single scenario and multiple damage scenario, are considered for comparative study in each examples. The obtained results from the numerical studies indicate that the MO algorithms are viable to the problem of damage detection in frame structures.

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