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DAMAGE IDENTIFICATION OF TRUSSES BY FINITE ELEMENT MODEL UPDATING USING AN ENHANCED LEVENBERG-MARQUARDT ALGORITHM

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ABSTRACT

This paper presents an efficient method for updating the structural finite element model. Model updating is performed through minimizing the difference of recorded acceleration of real damaged structure and hypothetical damaged structure, by updating physical parameters in each phase using iterative process of Levenberg-Marquardt algorithm. This algorithm is based on sensitivity analysis and provides a linear solution for nonlinear damage detection problem. The presented method is capable of detecting the exact location and ratio of structural damage in the presence of noise or incomplete data.

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KEY WORDS: Damage Identification; Model Updating; Levenberg-Marquardt Algorithm; Truss

1. INTRODUCTION

Within the structural health monitoring subject, detection and localization of damage is currently of growing interest among researchers [1]. The detection methods generally can be divided into two categories [2], the methods which are based upon static responses [3-5] and the other methods which use dynamic data [6-11]. The prevailing interest among researchers is focused on damage detection using vibration data [12, 13].

Since the damage has a nonlinear characteristic, the direct solution of the resulted system of equations is limited or maybe impossible. This set of equations should be treated by

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numerical methods through an iterative process. Current methods of damage detection are mostly composed of updating finite element model through minimizing the difference between responses of actual damaged structure and hypothetical damaged structure. Some of the mentioned methods apply the explorer optimization algorithms to update the structural model during an iterative process [14-18] . The other methods based on sensitivity analysis, utilize various algorithms to update the finite element model by minimizing the objective function.

Teughels and roeck [19] presented two sensitivity-based algorithms, modified Gauss-Newton and coupled local minimizers (CLM), for updating the finite element model. They detected the location and ratio of damage in bridges by model updating through minimization of the difference between frequencies and mode shapes.

Bakir et al [20] implemented another sensitivity-based method, trust region algorithm, for updating the finite element model. They obtained the place and damage ratio in reinforced concrete frames by minimizing the differences between the eigenfrequency and eigenmodes residuals. Their algorithm is also efficient where there is noise data.

Wu and Li [21] applied a two-stage sensitivity-based model updating procedure which was developed for structural health monitoring of IASC-ASCE benchmark steel structure. They concluded that the best estimation for updating parameters can be reached by using the Bayesian estimation technique.

Esfandiari et al [11] updated the structure model using a least square algorithm with appropriate stabilization method. Their method can detect the ratio and location of damage in trusses, where the noise exists in the frequency response function.

Lee [22] presented a method for identifying multiple cracks in a beam using the Newton-Raphson method, sensitivity analysis and natural frequencies. In his method, he used the natural frequencies of damaged structure.

However various methods have been developed for model updating, but still the novel ideas are being presented. The main goal of this paper is to develop an efficient method for updating the structural finite element model using Levenberg-Marquardt algorithm. This algorithm is based on sensitivity analysis and provides a linear solution for nonlinear damage detection problem. The presented method is capable of detecting the exact location and ratio of structural damage in the presence of noise or incomplete data.

This paper is organized as follows. First the fundamentals of damage detection are reviewed. Then, the formulation of Levenberg-Marquardt algorithm for model updating and the damage detection method are presented. Finally, results of the numerical simulations of three space structures are discussed and the efficiency of the proposed approach is investigated.

2. FUNDAMENTALS OF STRUCTURAL HEALTH MONITORING

The main idea of updating methods for damage detection is grounded on the fact that changes in structural response are due to changes in physical properties. In other words, during a reversed model updating the detailed changes of physical model can be found using the known responses. The registered responses of damaged structure are functions of the structural damage. It means that a specific damage causes a unique response. Hence one can search for this specific damage by knowing its associated response. The damage has a nonlinear characteristic however it can be simulated bay changes in structural parameters such as Young modulus or section area of members. The damage equation can be stated by a residual function:

$$r = R(X) - R_d \quad \text{(1a)}$$

$$X = \{x_1, x_1, ..., x_n\}\ ,\ \ 0 \le x_i \le 1 \quad \text{(1b)}$$

Where r is the residual function, $\boldsymbol{R}(x)$ and $\boldsymbol{R}_d$ are the response vectors of hypothetically damaged structure and existing damaged structure respectively. X represents the damage vector which consists of all structural members' damage (x_i) and n is the number of members. The goal of damage detection is to find the damage vector X, using the response vector of damage structure.

3. UPDATING PARAMETER

The updating parameter is the unknown physical feature of the model. In this paper the ratio of updated modulus of elasticity (E^e) to its initial value (E_0^e) is considered as updating parameter. The dimensionless updating parameter or is defined as follows:

$$x^e = -\frac{E^e - E_0^e}{E_0^e} \Rightarrow E^e = E_0^e(1 - x^e) \quad \text{(2)}$$

There is a linear relation between the updating parameter (x^e) and stiffness matrix of structural members:

$$K^e = K_0^e(1 - x^e)\ \ and\ \ K = K^u + \sum_{e=1}^{n} K_0^e(1 - x^e) \quad \text{(3)}$$

Where K_0^e and K^e are respectively the initial and updated stiffness matrices. K is the global stiffness matrix of structure and K^u is the stiffness matrix of members of undamaged members. n is the number of members which are updated.

4. OBJECTIVE FUNCTION

The damage detection procedure using model updating is similar to identification of unknown parameters in an optimization problem. Hence the damage index of structural members is formulated as optimization parameters so that the updated model can be able to simulate the response of damaged structure. By determination of error between responses of healthy structure and updated structure, and minimization of objective function in each step of iterative process, x^e is updated.

Minimizing of the objective function is defined as a non-linear least square minimization problem that is given by sum of squared differences [23].

$$f(X) = \frac{1}{2}(R(X) - R_d)^T (R(X) - R_d) = \frac{1}{2}\sum_{j=1}^{m} r_j^2(X) = \left\| r(X) \right\|^2 \tag{4}$$

Where r is the residual value of damage function.

Model updating is carried out by minimizing the difference between the acceleration response of actual damaged structure and hypothetical damaged structure.

5. LEVENBERG-MARQUARDT ALGORITHM

The presented method solves the nonlinear least square problem Eq. (4) using a sensitivity based optimization procedure. Gradient and hessian Matrices for optimization of objective function are defined as follows:

$$\nabla f(X) = J(X)^T r \tag{5a}$$

$$\nabla^2 f(X) = J(X)^T J(X) + Q(X) \approx J(X)^T J(X) \tag{5b}$$

Where J is the Jacobian matrix of first order partial derivatives of residuals. The hessian matrix is reformed in Levenberg-Marquardt algorithm [*24*].

$$\nabla^2 f(X_i) = [J(X_i)^T J(X_i) + \lambda\, diag(J(X_i)^T J(X_i))] \tag{6}$$

$$X_{i+1} = X_i - (\nabla^2 f(X_i))^{-1} \nabla f(X_i) \tag{7}$$

Updating parameter is searched through an iterative process proposed by Marquardt, and the structural response is simulated and the objective function is minimized. In the Eq. (6), λ is the damper factor and is assumed as a small value in first step. If the error rate reduces compering to previous step, λ decreases by a constant coefficient and if the error rate increases, λ increases with the same coefficient. Eventually the updating process using Levenberg-Marquardt algorithm is summarized as follows:

Updating begins using the formula (7).

Error rate (objective function) is calculated.

If the error rate increases, the λ Coefficients in Eq (6) increases and the error rate is calculated. This process continues until the error is reduced.

λ Coefficient is reduced and the process is resumed.

Since hessian in the algorithm is proportional to f variation, a correct path is traveled to reach the solution. In fact, in the direction of low curvature (for example, a relatively flat land), the path is covered with a long step and in the direction of high curvature (e.g. valleys) the path is covered with a short step. In this way, damage detection is properly solved. The only deficiency of this method is that in updating process, hessian matrix may not be invertible. In this paper, we use pseudo-inverse singular value decomposition to treat this issue.

6. JACOBIN MATRIX IN DAMAGE DETECTION OF PROBLEM

The Jacobian or Sensitivity matrix is the residual first-order derivative (r) with respect to vector of updating parameters (X). Sensitivity matrix is obtained from the following equations:

$$S = \frac{\partial r}{\partial X} = [S_1 ,\ldots, S_n] \tag{8}$$

$$S_j = \frac{R_{dj} - R_h}{\Delta X}, \quad R_{dj} = R(X_{dj}) \tag{9a}$$

$$X_{di} = \{0, \ldots, \Delta X_j, \ldots, 0\} \tag{9b}$$

According to the above mentioned equations, the sensitivity matrix is obtained by forward difference technique [25]. In these equations R_{dj} is the response of artificial damage of the jth member, R_h is the response vector of healthy structure and n is the number of members. It should be noted that for the solution of equations using the Gradient optimization methods, the number of equations should be greater than the number of known variables. Here the considered structural response is the recorded acceleration of some nodes under the impact load which is applied on the load during 0.005 seconds. Therefor the number of equations is sufficient to solve the system of equations.

$$S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_l \\ \vdots \\ s_{Nsensor} \end{bmatrix} \tag{10a}$$

$$A_l = \begin{bmatrix} a_{t_1} \\ a_{t_2} \\ \vdots \\ \vdots \\ a_{t_{nt}} \end{bmatrix} s_l = \begin{bmatrix} \frac{\partial a_{t_1}}{\partial x_1} & \frac{\partial a_{t_1}}{\partial x_2} & \cdots & \cdots & \frac{\partial a_{t_1}}{\partial x_n} \\ \frac{\partial a_{t_2}}{\partial x_1} & \frac{\partial a_{t_2}}{\partial x_2} & \cdots & \cdots & \frac{\partial a_{t_2}}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial a_{t_{nt}}}{\partial x_1} & \frac{\partial a_{t_{nt}}}{\partial x_2} & \cdots & \cdots & \frac{\partial a_{t_{nt}}}{\partial x_n} \end{bmatrix} \tag{10b}$$

A_l is the response acceleration vector at the *l-th* degree of freedom (sensor locations). The dimension of A_l is $nt\times 1$ and *nt* is the number of recorded accelerations steps. *n* is the number of structural parameters that must be known. Therefor the number of sensitivity matrix columns is equal to the number of components of updating parameter vector and the number of its rows is equal to the total number of recorded responses by sensors.

7. SINGULAR VALUE DECOMPOSITION

One of the most important decomposition tools to facilitate the solution of large linear system of equations is the singular value decomposition method [26] In this method, each matrix with any dimension can be converted to the multiply of three matrices that one of them is diagonal and the other two are Orthogonal matrices. This method is also one of the diagonalization methods. In this method a specific matrix is decomposed according to the following equation:

$$M = U\Sigma V^T = [u_1 \quad \dots \quad u_{nr}] \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 & 0 \\ 0 & \sigma_2 & 0 & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & 0 \\ 0 & \vdots & 0 & \ddots & 0 \\ 0 & \vdots & \cdots & 0 & \sigma_n \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1^T \\ \vdots \\ v_m^T \end{bmatrix} \tag{11}$$

The decomposed matrix dimension is $nr\times m$. $M\in R^{nr\times m}$, $U=(u_1 \ldots u_{nr})\in R^{nr\times nr}$, $V=(\upsilon_1 \ldots \upsilon_{nr})\in R^{m\times m}$, are square matrices. The columns of *U* and *V* matrices are called left-singular and right-singular vectors respectively. Σ is the diagonal matrix with singular values. The left-singular and right-singular vectors and the diagonal matrix are obtained as follows:

- The left-singular vectors of *M* are eigenvectors of $(M{*}M^T)$.
- The right-singular vectors of *M* are eigenvectors of $(M^T {*}M)$.
- The non-zero-singular values of *M* are the square roots of the non-zero eigenvalues of both $(M * M^T)$ and $(M^T *M)$.

According to the decomposition resulted from Eq. (11), the pseudo-inverse matrix is calculated using following equation.

$$M^+ = V\,\Sigma^+ U^T \tag{12}$$

$\sum^+$ can be easily calculated by the above mentioned explanations:

$$\Sigma^{+} = \begin{bmatrix} \frac{1}{\sigma_1} & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{\sigma_2} & 0 & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & 0 \\ 0 & \vdots & 0 & \ddots & 0 \\ 0 & \vdots & \cdots & 0 & \frac{1}{\sigma_n} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{13}$$

8. NOISE MODELING TO MEASURED DATA

The responses of damaged structures are measured by sensors in a laboratory or from actual structure. Therefore there is a noise in measured data sue to sensors. Since in the present study the responses of damaged structure are obtained from the analysis of numerical model, the noise should be considered in responses. This error is called measurement error and is applied to the responses by following equation [27]

$$a_{measured} = a_{calculated} + E_p \times N_{noise} \times a_{calculated} \tag{14}$$

Where $a_{measured}$ is the acceleration response vector with error and $a_{calculated}$ is the acceleration response vector calculated from Damage structure, E_p is the Noise level (e.g. 1percent, 5percent,…). N_{noise} is the normal distribution vector with mean zero and unit standard deviation.

9. DMAGE DETECTION

Optimization of the objective function with numerical methods begins form a starting point (Initial value variables vector) till another point is obtained and optimizing continues with repeating this process to the final point.

The steps of optimizing with an effective method for structural damage detection are presented as follows:

- Proper choice of the starting point and the control parameters of the algorithm,
- Physical properties of healthy structure are the starting point to update the finite element model.
- Selecting the appropriate factor λ is important and it differs for different cases.
- Updating the sensitivity matrix
- Updating the damage index
- Damage index using the suggested relation- ship Levenberg - Marquardt is updated. To stabilize the updating process, singular value decomposition is used:

$$X_{i+1} = X_i + H^{+} S(X_i)^T (R_d - R(X_i)) \tag{15a}$$

$$H^{+} = [S(X_i)^T S(X_i) + \lambda\, diag(S(X_i)^T S(X_i))]^{+} \tag{15b}$$

$$X_{i+1} = X_i + (V\, \Sigma^{+} U^T)\; S(X_i)^T r_i \tag{15c}$$

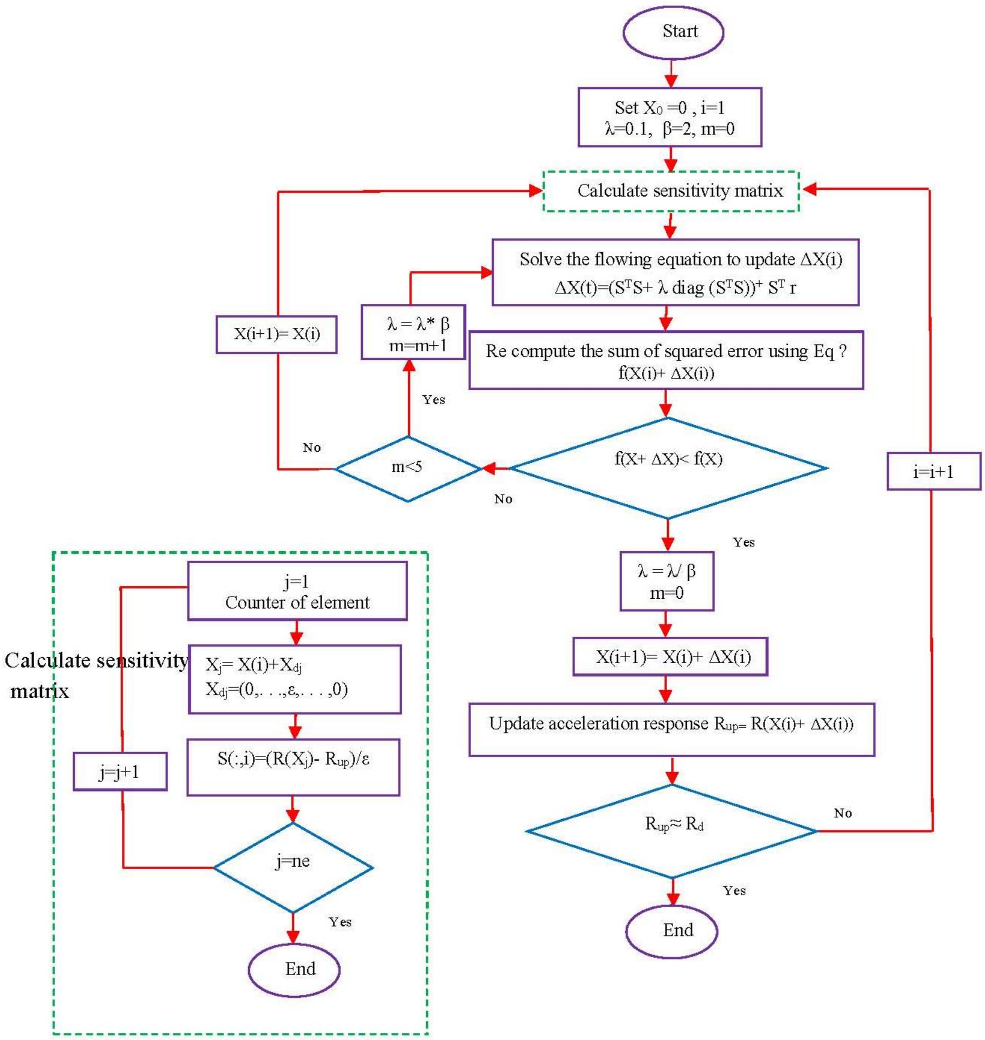


Figure 1. The flowchart for presented method

- Updating the objective function
- If the value of objective function is reduced it is accepted.
- If the objective function value is more than the previous value, a new value is ascribed to the λ and the value of X is calculated again.
- Using the above relations and new damage index, the objective function is again updated value is calculated.

- If the value of the objective function decreases, the updated damage index is acceptable.
- The recurrent updating of the damage index to the decrease of objective function value will continue for 5 steps.
- If the updated damage index doesn't change the objective function, updating process will continue with the failure index of previous stage
- To continue updating in the next stage, λ value decreases.

The steps are given in the following flowchart:

10. NUMERICAL RESULTS

To demonstrate the efficiency of the Levenberg-Marquardt algorithm in solving complex discrete structures, three space-structures are damage detected. Using the acceleration response recorded in some points, structure damage detection is done by extension of Levenberg – Marquardt algorithm in updating the structure models. The Riley damping has been used to model the structural damping. Triangular impulsive load in time step 0.005sec is induced vertically on the structure nodes.

Since the acceleration of the damaged structure is measured in laboratory, they are the origin of the errors related to Eq. (14).

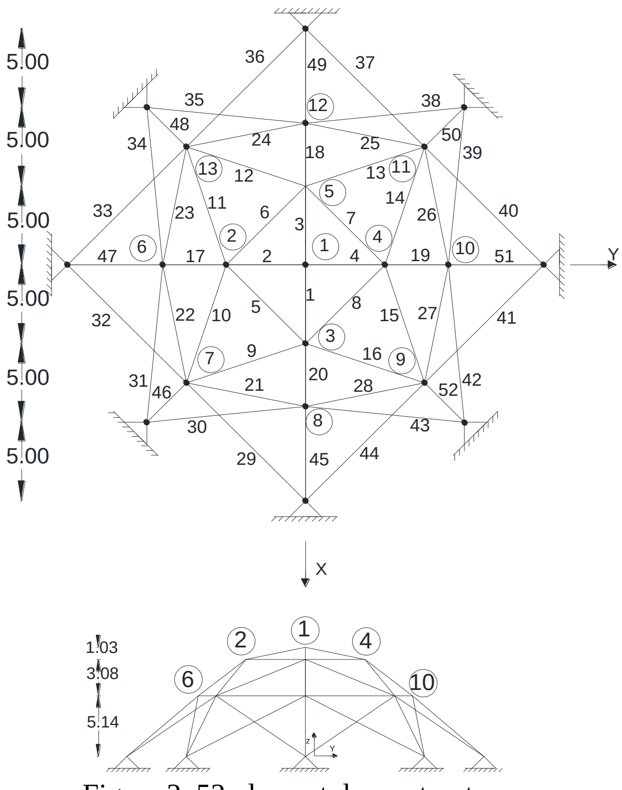


Figure 2. 52-element dome structure

10.1 SPACE STRUCTURE 52-MEMBERS DOME

A fifty-two member- dome as shown in Figure 2 is considered here to confirm the proposed method. Finite element model of the structure is composed of 21 nodes with 39 active degrees of freedom.

In this example, according to the reference, some sensor patterns have been considered as shown in figure 3. Also, the number of the nodes and members are shown in figure 2. Various conditions and pattern of damage are given in Tables 1 and 2 respectively. In figure 3.a and b, the sensors record the responses in three directions and in case c sensors are able to record the response in one direction. In case c, $\otimes$ shows the downward direction and in case d, it’s assumed that sensors are located for all degrees of freedom.

Table 1: Different condition for 52-element dome

Condition	**Scenario**	**Sensor pattern**	**Noise level**
A	1	d	0
B	2	a	3%
C	2	b	1%
D	3	a	3%
E	4	b	3%
F	4	c	1%

Table 2: Different damage scenarios 52-element dome

Scenario	**Number of element**	**Damage ratio**
1	27	10%
	34	10%
2	22	30%
	44	30%
3	2	30%
	10	30%
	30	30%
4	9	40%
	13	30%
	40	30%

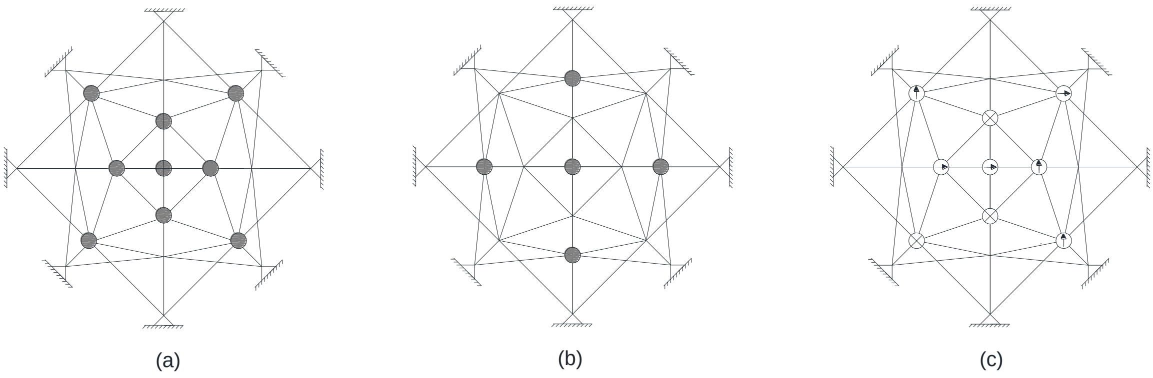


Figure 3. The various Position of sensors in 52-element dome structure

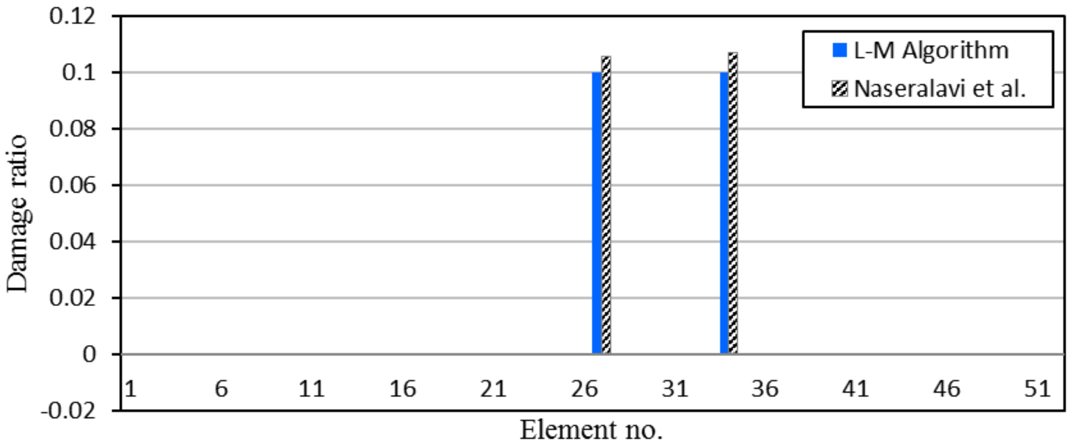


Figure 4. Damage detection result for 52-element dome structure of the condition A

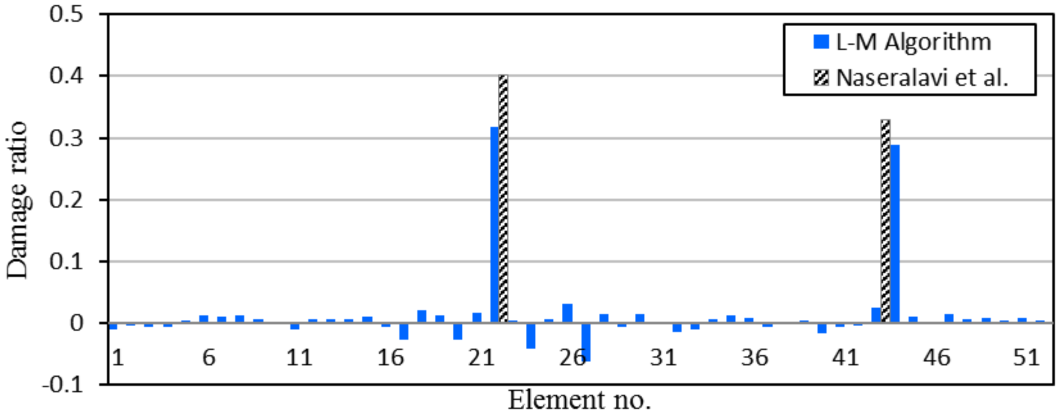


Figure 5. Damage detection result for 52-element dome structure of the condition B

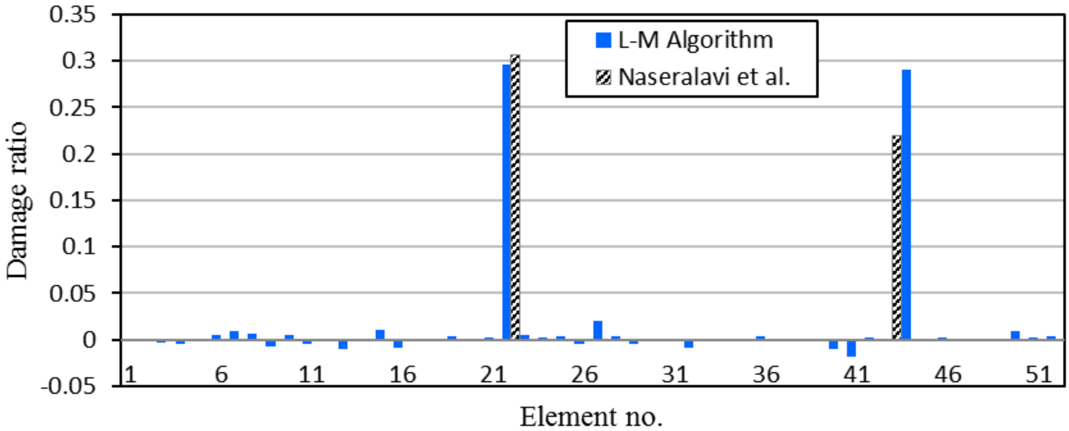


Figure 6. Damage detection result for 52-element dome structure of the condition C

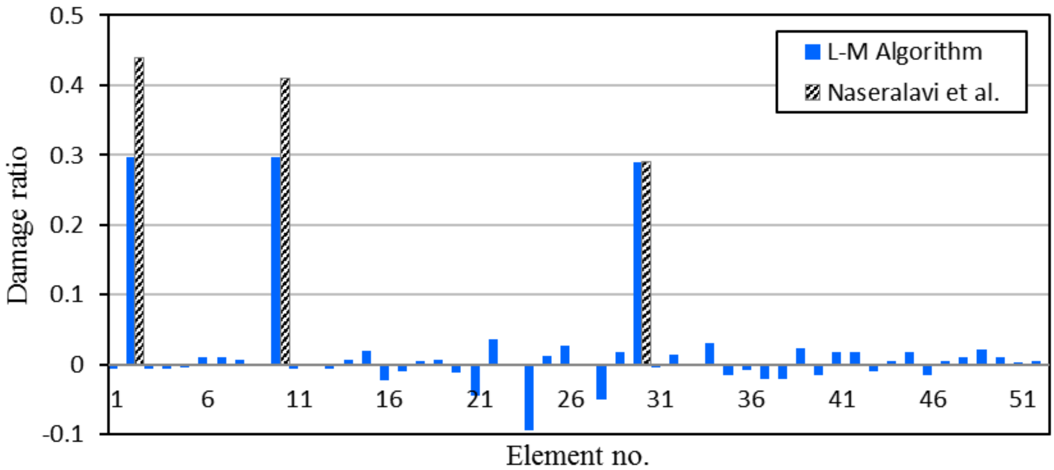


Figure 7. Damage detection result for 52-element dome structure of the condition D

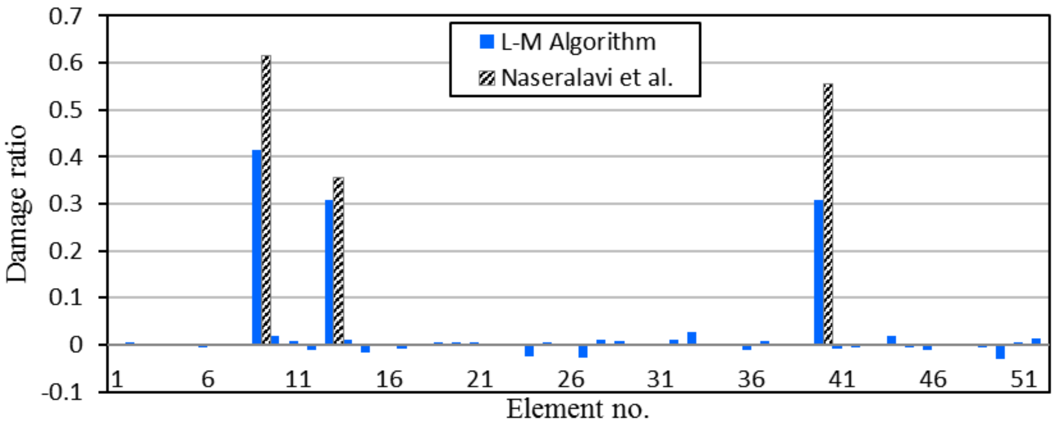


Figure 8. Damage detection result for 52-element dome structure of the condition E

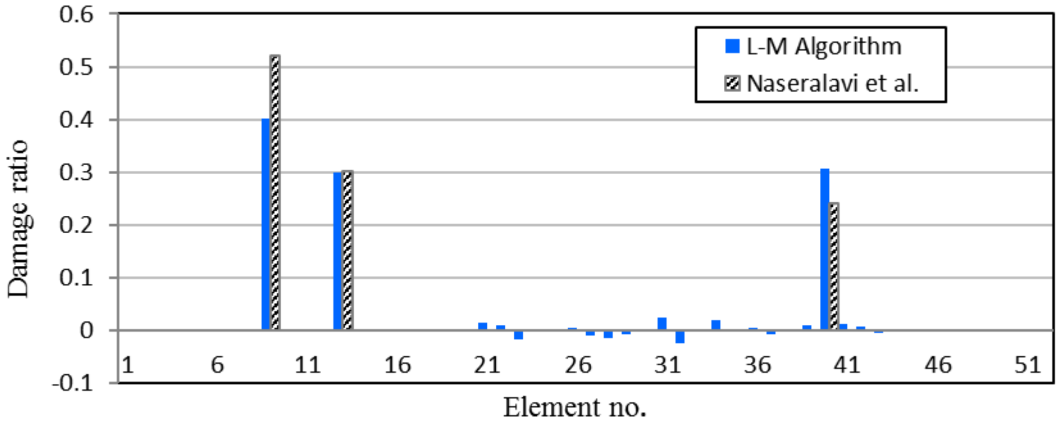


Figure 9. Damage detection result for 52-element dome structure of the condition F

10.2 Space Structure 120- member Dome

A 120 member- dome as shown in Figure 10 is considered here to confirm the proposed method.

The section area of members of a 120-member dome is optimized under static loading [29] Physical model of the structure consists of 49 nodes with 117 active degrees of freedom. Number of the points and members are shown in Figure 10. Various conditions and damage pattern are given in Tables 3 and 4 respectively.

Table 3: Different condition for 120-element dome

Condition	Scenario	Sensor pattern	Noise level
A	2	a	0
B	2	b	0
C	4	a	0
D	4	b	0
E	3	b	%5
F	1	a	%2

Table 4: Different damage scenarios 120-element dome

Scenario	Number of element	Damage ratio
1	18	%10
	51	%40
	52	%30
	111	%25
	118	%5
2	16	%30

	27	%12
	41	%32
	49	%5
	62	%40
	96	%25
	117	%10
3	1	%25
	34	%30
	42	%40
	56	%5
	58	%10
	65	%35
	67	%25
	77	%17
	91	%9
4	6	%25
	22	%25
	32	%40
	34	%5
	35	%40
	40	%30
	91	%17
	92	%12
	99	%10
	112	%11
	119	%9

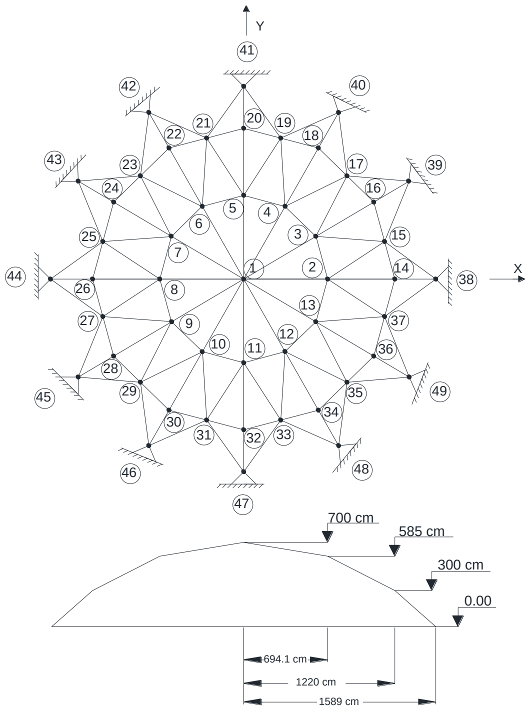
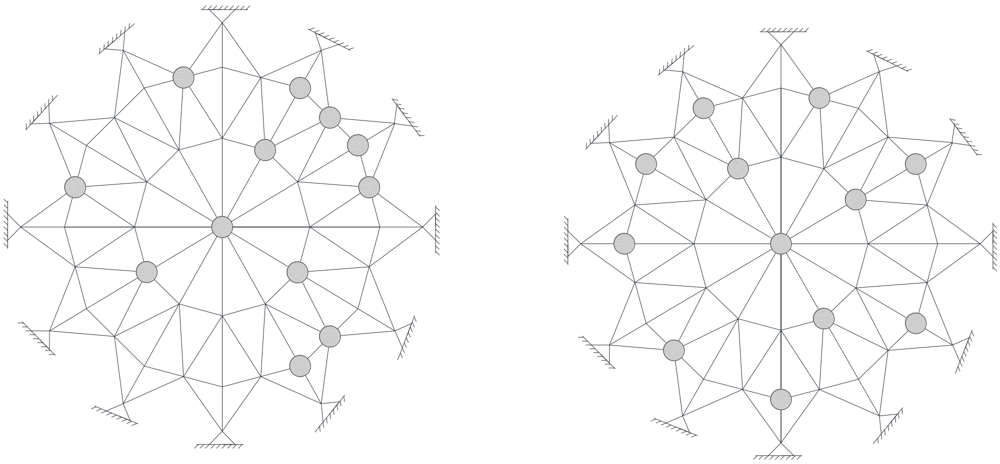


Figure 10. 120-element dome structure



(a) (b)

Figure 11. The various Position of sensors in 120-element dome structure

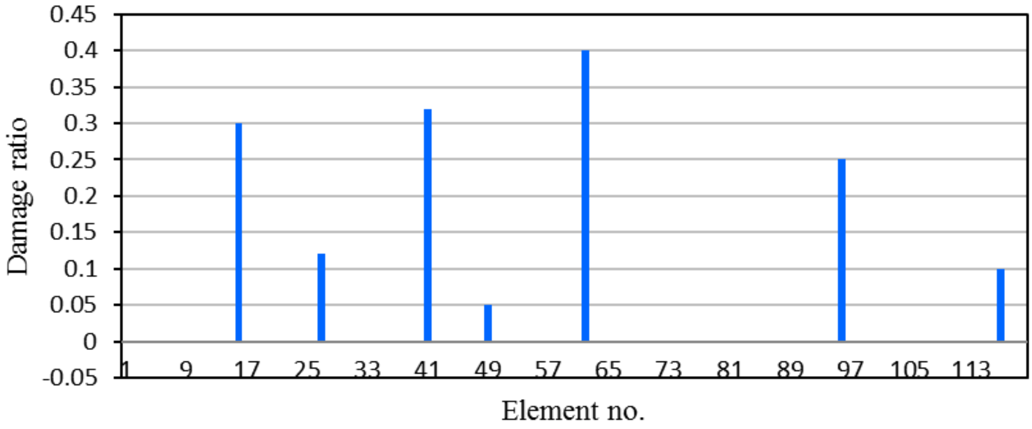


Figure 12. Damage detection result for 120-element dome structure of the condition A

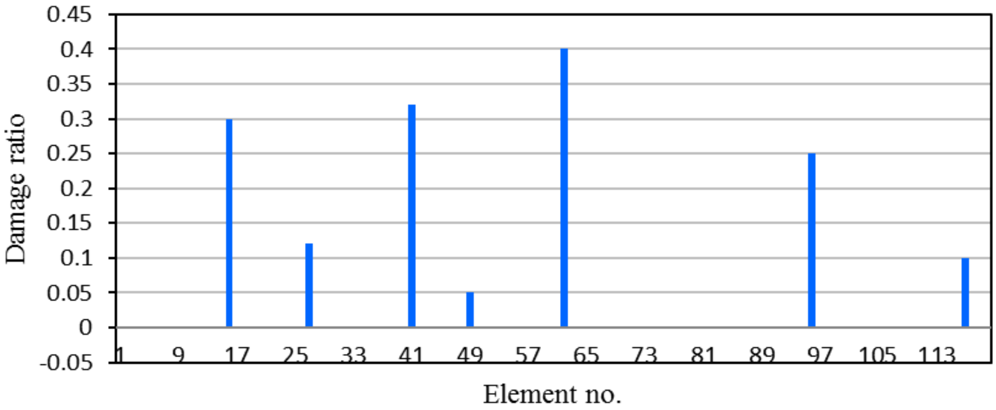


Figure 13. Damage detection result for 120-element dome structure of the condition B

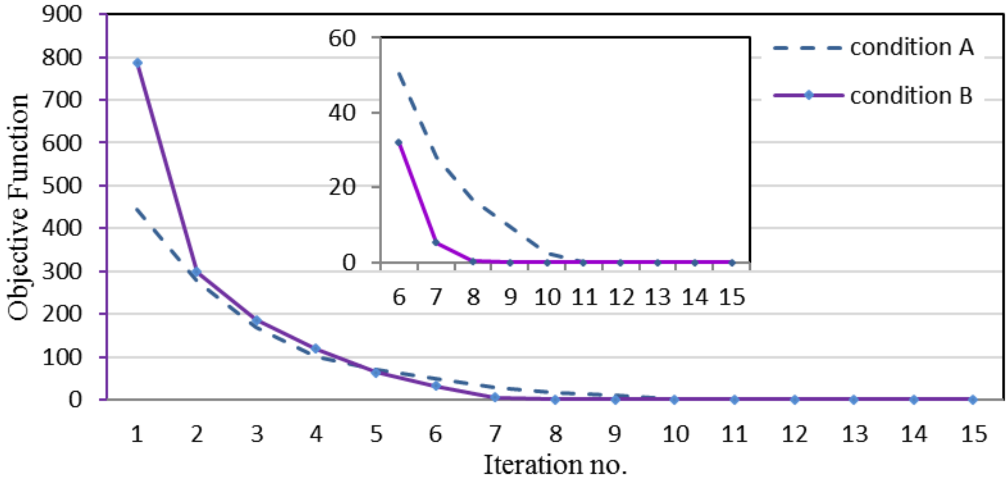


Figure 14.Convergence diagram for 120-element dome structure of the condition A&B

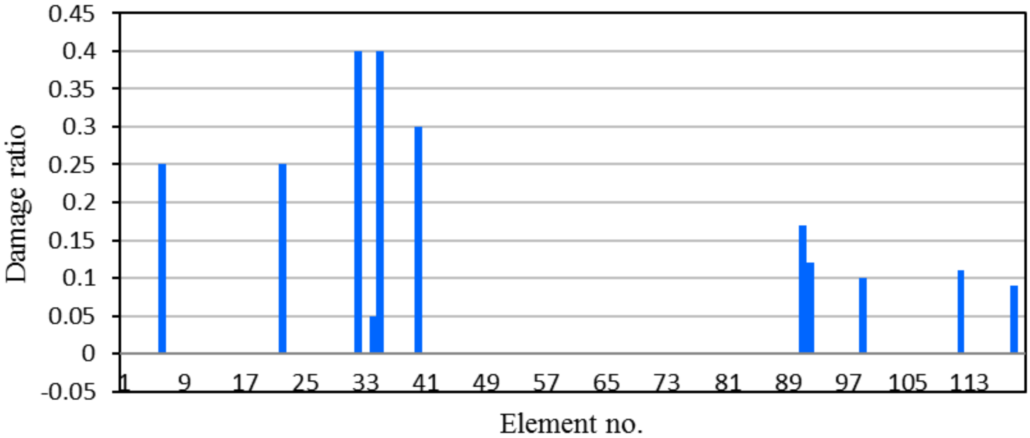


Figure 15. Damage detection result for 120-element dome structure of the condition

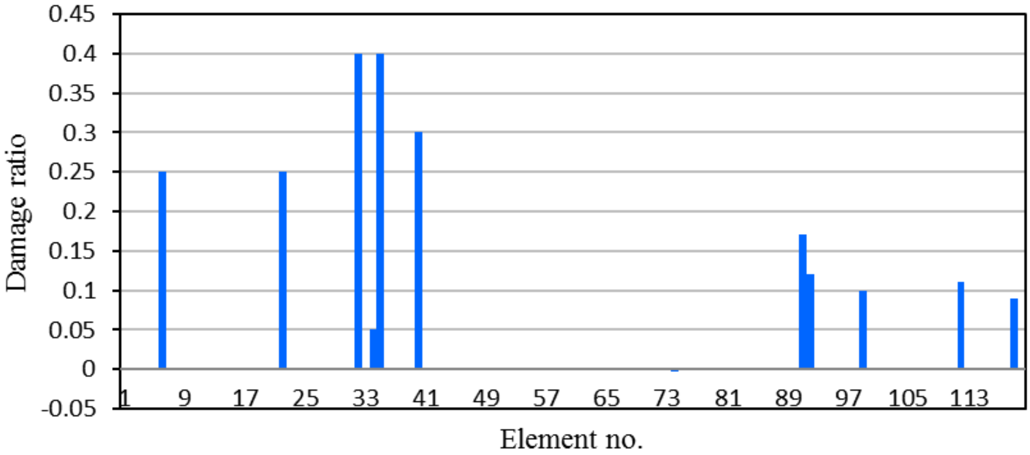


Figure 16. Damage detection result for 120-element dome structure of the condition D

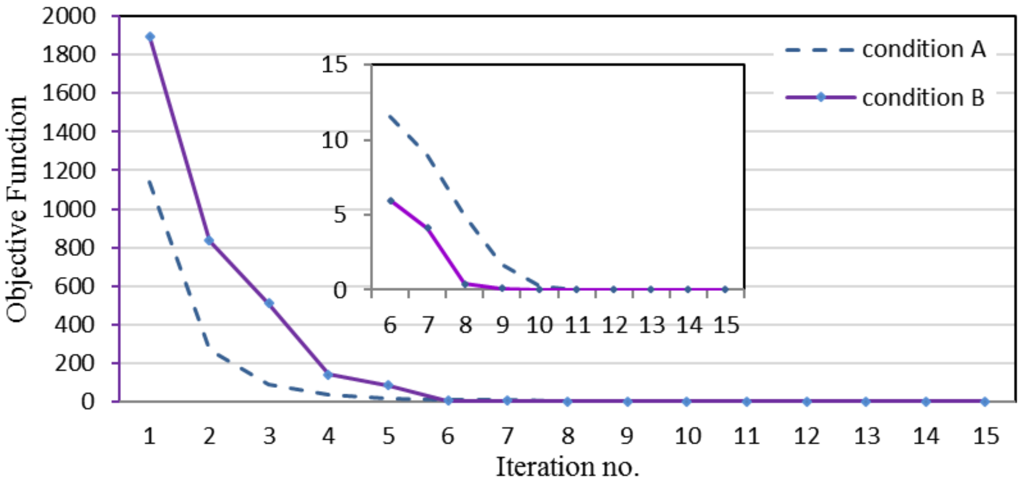


Figure 17.Convergence diagram for 120-element dome structure of the condition C&D

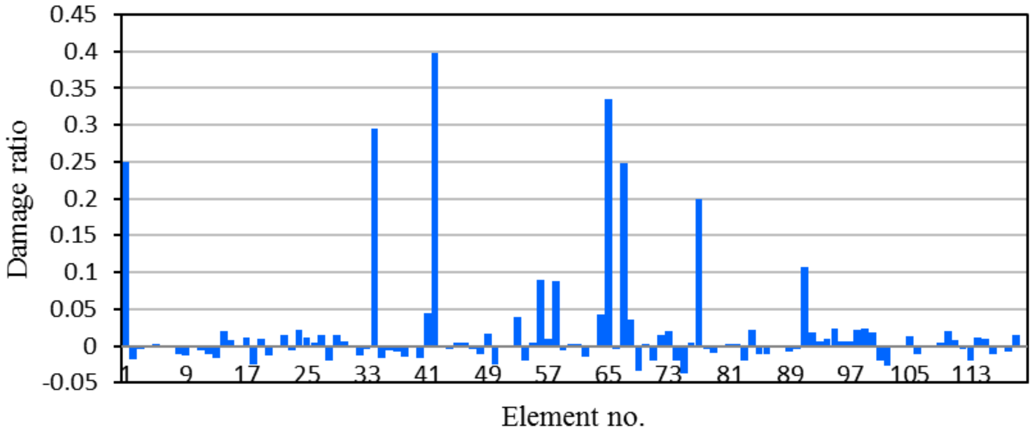


Figure 18. Damage detection result for 120-element dome structure of the condition E

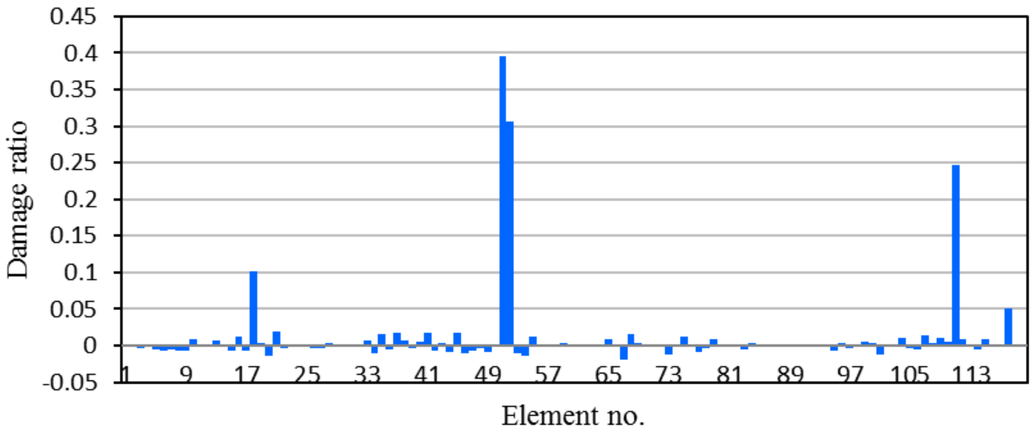


Figure 19. Damage detection result for 120-element dome structure of the condition F

10.3 A 800 Element Double Layer Grid

For more investigation, the proposed algorithm is implanted on a full-member structure for damage detection. Physical model of the structure is composed of 221 nodes and 555 active degrees of freedom. Modulus of elasticity and the weight per unit volume are respectively 210000 Mpa and 7850 Kg/m^3.

The number of points and members are shown in Figure 20. Patterns and various conditions of damage are respectively given in Tables 5 and 6. In this example, two sensors pattern are randomly considered as shown in Figure 21. Also, two pattern loads are applied on the structure in 5 points of in the first pattern and one point in the second pattern.

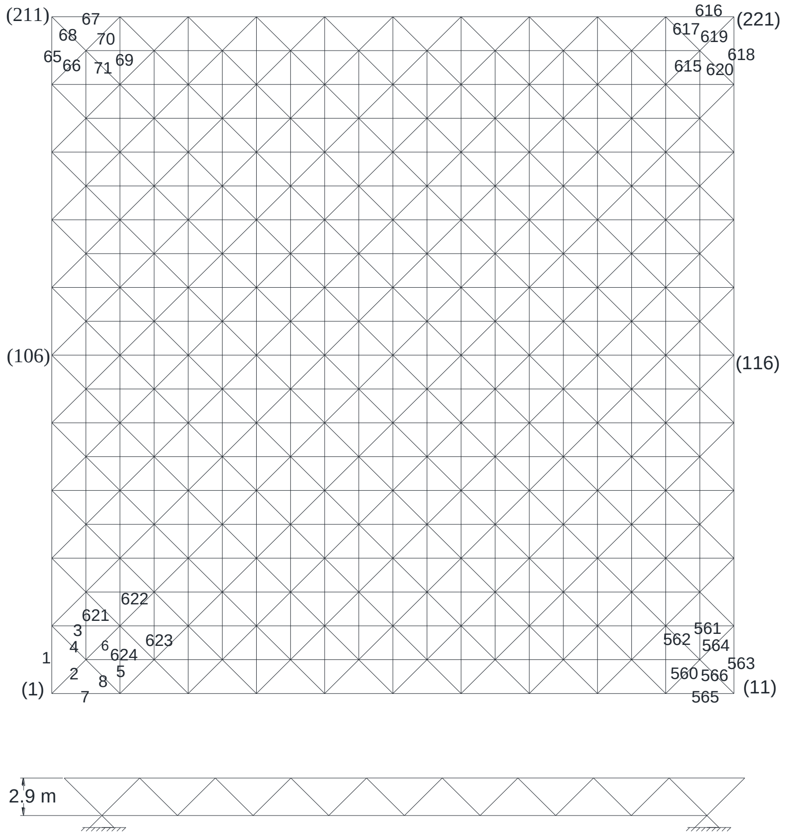


Figure 20. 800-Element double layer grid

Table 5: Different damage scenarios 120-element dome

Scenario	Number of element	Damage ratio
1	12	15%
	35	25%
	50	8%
	94	24%
	124	17%
	202	23%
	241	15%
	245	31%
	264	30%
	293	17%
	306	25%
	377	37%
	422	11%
	455	40%
	465	17%

	483	27%
	570	33%
	682	15%
	687	8%
	750	22%
2	32	26%
	48	16%
	238	11%
	288	40%
	296	25%
	339	32%
	356	15%
	535	31%
	668	5%
	737	12%
3	150	15%
	327	25%
	438	27%
	471	9%
	472	32%
	521	12%
	531	30%
	538	5%
	718	17%
	793	40%
4	154	40%
	273	11%
	417	15%
	486	31%
	581	25%
	707	8%
	748	17%

Table 6: Different condition for 120-element dome

Condition	Scenario	Sensor pattern	Loading Pattern	Noise level
A	1	b	1	1%
B	2	a	1	0
C	3	a	1	0
D	3	a	2	0
E	3	b	2	1%
F	4	a	1	2%

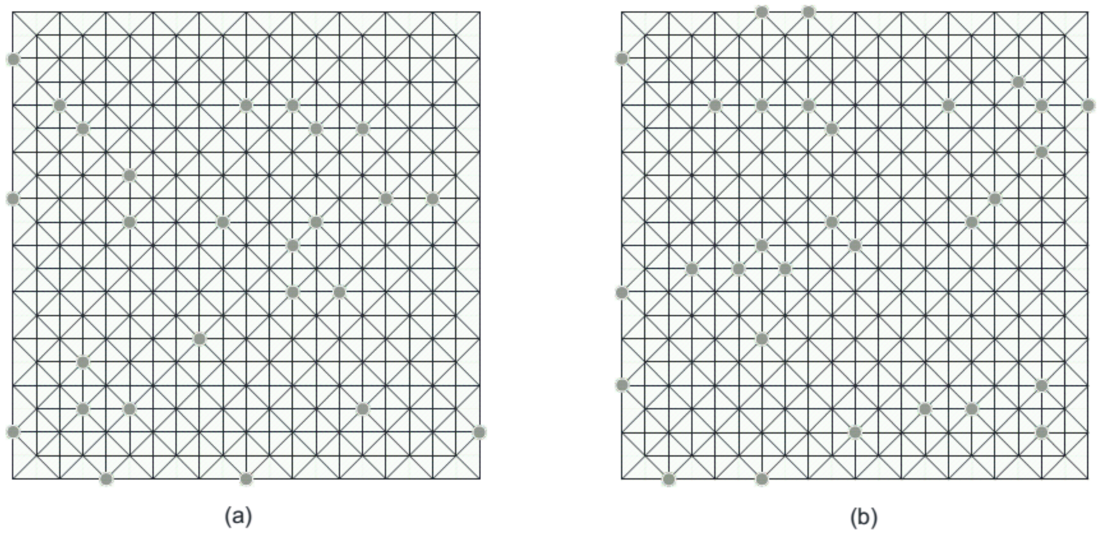


Figure 21. The various Position of sensors in 800-element double layer grid

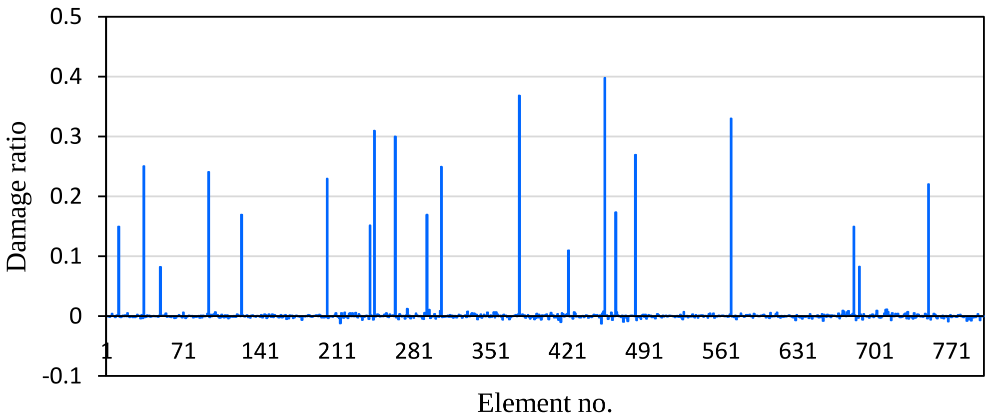


Figure 22. Damage detection result for 800-element double layer grid of the condition A

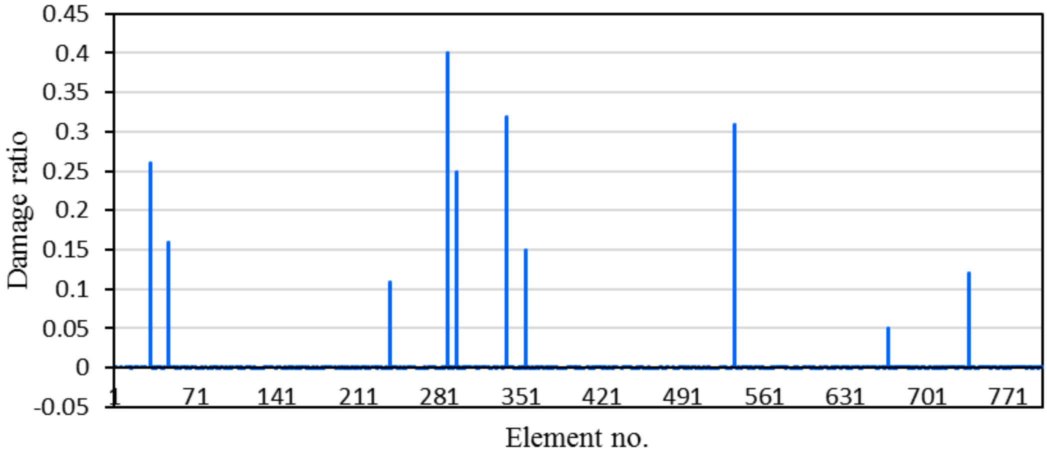


Figure 23. Damage detection result for 800-element double layer grid of the condition B

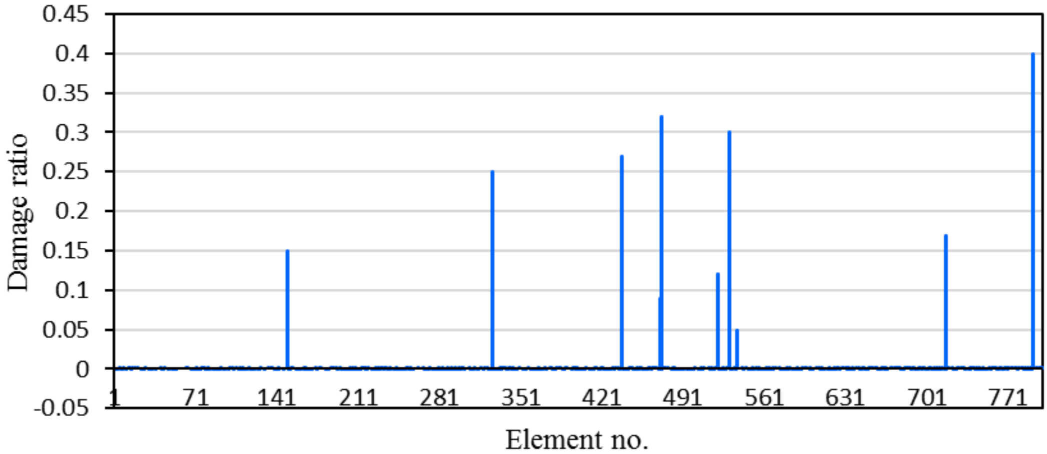


Figure 24. Damage detection result for 800-element double layer grid of the condition C

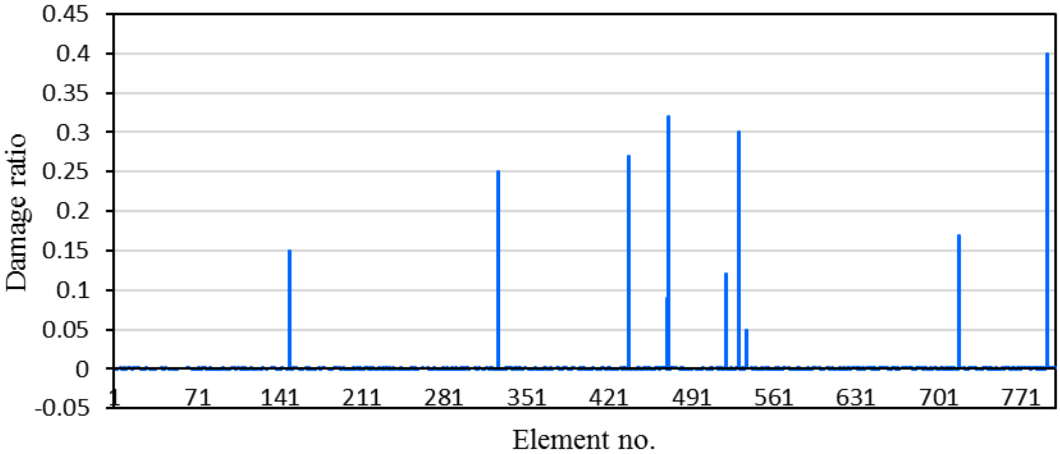


Figure 25. Damage detection result for 800-element double layer grid of the condition D

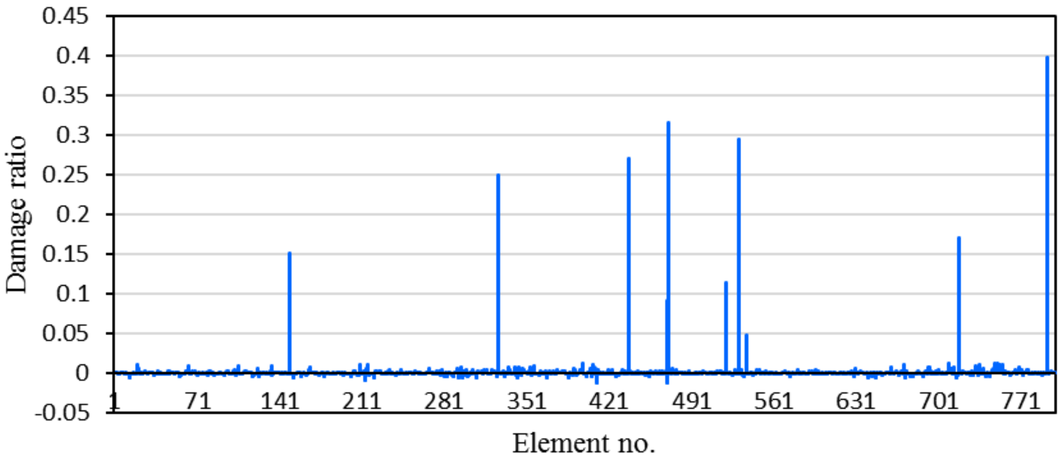


Figure 26. Damage detection result for 800-element double layer grid of the condition E

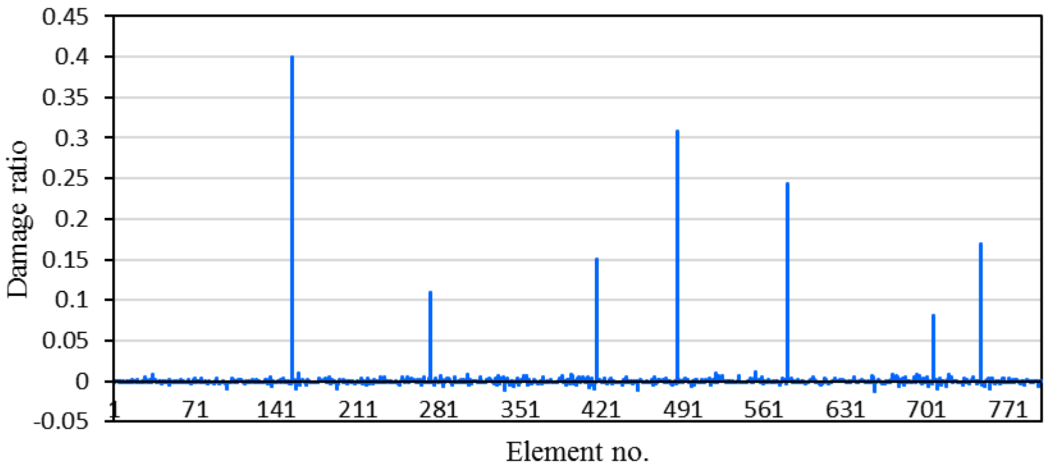


Figure 27. Damage detection result for 800-element double layer grid of the condition F

11. CONCLUSION

This work proposes damage identification of truss structures using the Levenberg–Marquardt algorithm based on sensitivity analysis. Due to the high cost of data-receiving systems (including installation of sensors and instruments for keeping them), the proposed method can effectively detect damage with the assumption of putting limited number of sensors in structure and therefore it has more advantages in this respect.

Since sensor placement is carried out randomly, the results show that the proposed method with different sensor placement patterns is able to detect the damage, but it should be noted the location of sensors is significant in terms of precision and convergence speed. From different patterns of sensors placement it is concluded that the quantity of damage with symmetrical sensors in structures is detected with more high accuracy and speed.The results show that the sensitivity of the method is higher compared to the little acceleration change and even in big structures the gradual loading in one of the points has been sufficient and is able to detect the damage with precision and speed.

To investigate and compare the performance of Levenberg-Marquardt method with other methods some examples were solved to detect damage in full-member discrete structures. The obtained numerical results are then compared with available numerical methods, and excellent agreements are found.

Also the results from problem damage detection of 800-element double layer grid showed that the proposed method has efficiency and high precision for the damage detection in large scale structures and it can be used in practical issues.

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